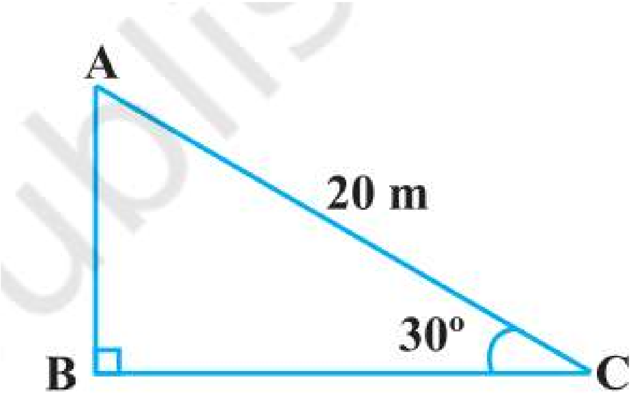
Chapter 9: Some Applications of Trigonometry

Class 10 Math Chapter 9 Solutions (English Medium)

Exercise 9.1

Q 1. **A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30° (see Fig. 9.11).**



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Let AB be the vertical pole and AC be the tightly stretched rope.

Given: Length of the rope (Hypotenuse, AC) = 20 m

Angle of elevation $\angle C = 30^\circ$

We need to find the height of the pole AB (Perpendicular).

In right $\triangle ABC$,

$\sin 30^\circ = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{AB}{AC}$

$\frac{1}{2} = \frac{AB}{20}$

$2 \times AB = 20 \Rightarrow AB = \frac{20}{2} = 10 \text{ m}$

Hence, the height of the pole is 10 m.

Q 2. **A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.**

Let BD be the original tree. It breaks at point A and the broken part AD bends to become AC, touching the ground at C.

So, total height of tree = $AB + AC$.

Given: $\angle C = 30^\circ$, and BC (Base) = 8 m.

In right $\triangle ABC$, to find AB (Perpendicular):

$\tan 30^\circ = \frac{\text{Perpendicular}}{\text{Base}} = \frac{AB}{BC}$

$\frac{1}{\sqrt{3}} = \frac{AB}{8} \Rightarrow AB = \frac{8}{\sqrt{3}}\ \text{m}$

To find AC (Hypotenuse):

$\cos 30^\circ = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{BC}{AC}$

$\frac{\sqrt{3}}{2} = \frac{8}{AC} \Rightarrow \sqrt{3} \times AC = 16 \Rightarrow AC = \frac{16}{\sqrt{3}}\ \text{m}$

Total height of the tree = $AB + AC = \frac{8}{\sqrt{3}} + \frac{16}{\sqrt{3}} = \frac{24}{\sqrt{3}}$

Rationalizing: $\frac{24}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{24\sqrt{3}}{3} = 8\sqrt{3}\ \text{m}$

Hence, the height of the tree is $8\sqrt{3}$ m.

Q 3. **A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m, and is inclined at an angle of 30° to the ground, whereas for elder children, she wants to have a steep slide at a height of 3 m, and inclined at an angle of 60° to the ground. What should be the length of the slide in each case?**

Case 1 (For children below 5 years):

Let height (Perpendicular) AB = 1.5 m, and slide length (Hypotenuse) be AC. Angle = 30°.

$\sin 30^\circ = \frac{AB}{AC}$

$\frac{1}{2} = \frac{1.5}{AC} \Rightarrow AC = 1.5 \times 2 = 3 \text{ m}$

Case 2 (For elder children):

Let height (Perpendicular) PQ = 3 m, and slide length (Hypotenuse) be PR. Angle = 60°.

$\sin 60^\circ = \frac{PQ}{PR}$

$\frac{\sqrt{3}}{2} = \frac{3}{PR} \Rightarrow \sqrt{3} \times PR = 6$

$PR = \frac{6}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3} \text{ m}$

Hence, the length of the slide should be 3 m and $2\sqrt{3}\text{ m}$ respectively.

Q 4. The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower, is 30°. Find the height of the tower.

Let AB be the height of the tower and C be the point on the ground.

Given: Distance BC = 30 m, and angle of elevation $\angle C = 30^{\circ}$.

In right $\triangle ABC$,

$\tan 30^{\circ} = \frac{AB}{BC}$

$\frac{1}{\sqrt{3}} = \frac{AB}{30}$

$AB = \frac{30}{\sqrt{3}}$

Rationalizing: $AB = \frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{30\sqrt{3}}{3} = 10\sqrt{3}\text{ m}$

Hence, the height of the tower is $10\sqrt{3}\text{ m}$.

Q 5. A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.

Let AB be the height of the kite = 60 m and AC be the length of the string.

Given: Angle of inclination $\angle C = 60^{\circ}$.

In right $\triangle ABC$,

$\sin 60^{\circ} = \frac{AB}{AC}$

$\frac{\sqrt{3}}{2} = \frac{60}{AC}$

$\sqrt{3} \times AC = 120$

$AC = \frac{120}{\sqrt{3}}$

Rationalizing: $AC = \frac{120}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{120\sqrt{3}}{3} = 40\sqrt{3}\text{ m}$

Hence, the length of the string is $40\sqrt{3}\text{ m}$.

Q 6. A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

Let the height of building PQ = 30 m and height of boy AB = 1.5 m.

Draw a horizontal line AR from the boy's eye level.

The part of building above eye level is PR = $30 - 1.5 = 28.5 \text{ m}$.

Let the boy walk from A to C.

Initial angle $\angle PAR = 30^\circ$, and final angle $\angle PCR = 60^\circ$.

In right $\triangle PCR$:

$$\tan 60^\circ = \frac{PR}{CR} \Rightarrow \sqrt{3} = \frac{28.5}{CR} \Rightarrow CR = \frac{28.5}{\sqrt{3}}$$

In right $\triangle PAR$:

$$\tan 30^\circ = \frac{PR}{AR} \Rightarrow \frac{1}{\sqrt{3}} = \frac{28.5}{AR} \Rightarrow AR = 28.5\sqrt{3}$$

Distance walked AC = $AR - CR$

$$AC = 28.5\sqrt{3} - \frac{28.5}{\sqrt{3}}$$

$$AC = \frac{28.5 \times 3 - 28.5}{\sqrt{3}} = \frac{85.5 - 28.5}{\sqrt{3}} = \frac{57}{\sqrt{3}}$$

Rationalizing: $\frac{57}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{57\sqrt{3}}{3} = 19\sqrt{3} \text{ m}$

Hence, the distance walked by the boy is $19\sqrt{3} \text{ m}$.

Q 7. From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.

Let the height of building BC = 20 m and height of transmission tower AB = h.

D is a point on the ground. Elevation angle to bottom of tower (B) is 45° and to top of tower (A) is 60°.

In right $\triangle BCD$:

$\tan 45^\circ = \frac{BC}{CD} \Rightarrow 1 = \frac{20}{CD} \Rightarrow CD = 20 \text{ m}$

In right $\triangle ACD$:

Perpendicular AC = $AB + BC = h + 20$

$\tan 60^\circ = \frac{AC}{CD} \Rightarrow \sqrt{3} = \frac{h+20}{20}$

$h + 20 = 20\sqrt{3}$

$h = 20\sqrt{3} - 20 = 20(\sqrt{3} - 1) \text{ m}$

Hence, the height of the tower is $20(\sqrt{3} - 1) \text{ m}$**.**

Q 8. **A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.**

Let the height of pedestal BC = h and the statue on top of it AB = 1.6 m.

D is a point on the ground. Elevation angle to top of statue (A) is 60° and to top of pedestal (B) is 45°.

In right $\triangle BCD$:

$\tan 45^\circ = \frac{BC}{CD} \Rightarrow 1 = \frac{h}{CD} \Rightarrow CD = h$

In right $\triangle ACD$:

Perpendicular AC = $AB + BC = 1.6 + h$

$\tan 60^\circ = \frac{AC}{CD} \Rightarrow \sqrt{3} = \frac{1.6+h}{h}$

$\sqrt{3}h = 1.6 + h \Rightarrow \sqrt{3}h - h = 1.6 \Rightarrow h(\sqrt{3} - 1) = 1.6$

$h = \frac{1.6}{\sqrt{3}-1}$

Rationalizing: $\frac{1.6}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{1.6(\sqrt{3}+1)}{3-1} = \frac{1.6(\sqrt{3}+1)}{2} = 0.8(\sqrt{3} + 1)\ \text{m}$

Hence, the height of the pedestal is $0.8(\sqrt{3} + 1)\ \text{m}$.

Q 9. **The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.**

Let the tower AB = 50 m and the building CD = h. Let distance between them BD = x.

Given: Angle of elevation of top of building from foot of tower $\angle CBD = 30^\circ$. Angle of elevation of top of tower from foot of building $\angle ADB = 60^\circ$.

In right $\triangle ABD$:

$\tan 60^\circ = \frac{AB}{BD} \Rightarrow \sqrt{3} = \frac{50}{x} \Rightarrow x = \frac{50}{\sqrt{3}}$ m

In right $\triangle CDB$:

$\tan 30^\circ = \frac{CD}{BD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x} \Rightarrow h = \frac{x}{\sqrt{3}}$

Substituting x: $h = \frac{50/\sqrt{3}}{\sqrt{3}} = \frac{50}{3} = 16\frac{2}{3}$ m

Hence, the height of the building is $16\frac{2}{3}$ m.

Q 10. **Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30°, respectively. Find the height of the poles and the distances of the point from the poles.**

Let AB and CD be the two poles of equal height h. Width of road BD = 80 m.

Let P be a point on the road. Let BP = x, then PD = $80 - x$.

Given: $\angle APB = 60^\circ$ and $\angle CPD = 30^\circ$.

In right $\triangle APB$:

$\tan 60^\circ = \frac{AB}{BP} \Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow h = x\sqrt{3}$ --- (1)

In right $\triangle CPD$:

$\tan 30^\circ = \frac{CD}{PD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{80-x} \Rightarrow h = \frac{80-x}{\sqrt{3}}$ --- (2)

Equating h from (1) and (2):

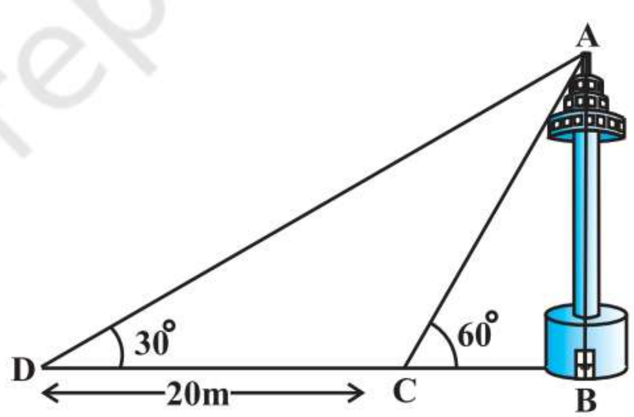
$x\sqrt{3} = \frac{80-x}{\sqrt{3}} \Rightarrow 3x = 80 - x \Rightarrow 4x = 80 \Rightarrow x = 20 \text{ m}$

Now, $h = x\sqrt{3} = 20\sqrt{3} \text{ m}$.

And distance from other pole = $80 - x = 80 - 20 = 60 \text{ m}$.

Hence, the height of poles is $20\sqrt{3} \text{ m}$ and the distances of the point are 20 m and 60 m.

Q 11. **A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30° (see Fig. 9.12). Find the height of the tower and the width of the canal.**



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Let tower AB = h and width of canal BC = x.

From point C, angle is 60°. From point D (20m away from C), angle is 30° (so CD = 20 m).

In right $\triangle ABC$:

$\tan 60^\circ = \frac{AB}{BC} \Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow h = x\sqrt{3}$ --- (1)

In right $\triangle ABD$:

Base BD = $BC + CD = x + 20$

$\tan 30^\circ = \frac{AB}{BD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+20} \Rightarrow h = \frac{x+20}{\sqrt{3}}$ --- (2)

From (1) and (2):

$x\sqrt{3} = \frac{x+20}{\sqrt{3}} \Rightarrow 3x = x + 20 \Rightarrow 2x = 20 \Rightarrow x = 10 \text{ m}$

Now $h = x\sqrt{3} = 10\sqrt{3}$ m.

Hence, the height of the tower is $10\sqrt{3}$ m and width of the canal is 10 m.

Q 12. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

Let building AB = 7 m and cable tower be CD. Draw horizontal line AE from A to tower CD.

Now tower has two parts: CE (above AE) and ED (below AE). Here ED = AB = 7 m.

Given: Elevation angle to top $\angle CAE = 60^\circ$ and depression angle to foot $\angle EAD = 45^\circ$.

In right $\triangle AED$:

$\tan 45^\circ = \frac{ED}{AE} \Rightarrow 1 = \frac{7}{AE} \Rightarrow AE = 7 \text{ m}$

In right $\triangle AEC$:

$\tan 60^\circ = \frac{CE}{AE} \Rightarrow \sqrt{3} = \frac{CE}{7} \Rightarrow CE = 7\sqrt{3} \text{ m}$

Total height of tower CD = $CE + ED = 7\sqrt{3} + 7 = 7(\sqrt{3} + 1) \text{ m}$

Hence, the height of the cable tower is $7(\sqrt{3} + 1) \text{ m}$.

Q 13. As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.

Let lighthouse AB = 75 m and two ships be at points C and D.

Angles of depression are 45° and 30°. (By alternate interior angles, $\angle ACB = 45^\circ$ and $\angle ADB = 30^\circ$).

In right $\triangle ABC$:

$$\tan 45^\circ = \frac{AB}{BC} \Rightarrow 1 = \frac{75}{BC} \Rightarrow BC = 75 \text{ m}$$

In right $\triangle ABD$:

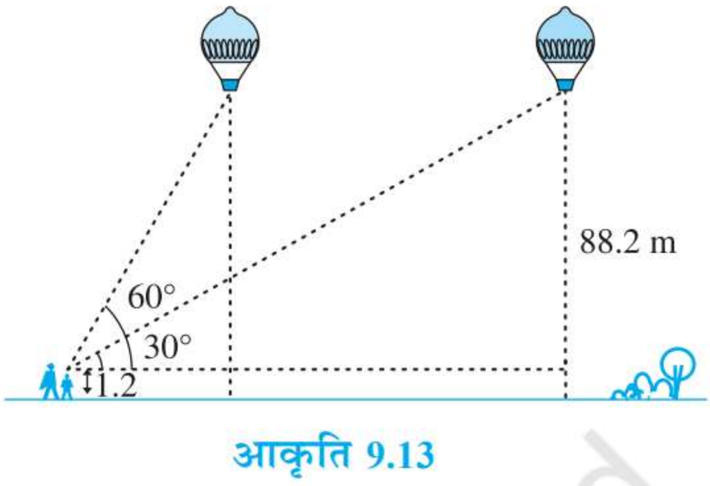
$$\tan 30^\circ = \frac{AB}{BD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{75}{BD} \Rightarrow BD = 75\sqrt{3} \text{ m}$$

Distance between two ships CD = $BD - BC$

$$CD = 75\sqrt{3} - 75 = 75(\sqrt{3} - 1) \text{ m}$$

Hence, the distance between the two ships is $75(\sqrt{3} - 1)$ m.

Q 14. **A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After some time, the angle of elevation reduces to 30° (see Fig. 9.13). Find the distance travelled by the balloon during the interval.**



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Height of girl = 1.2 m. Height of balloon from ground = 88.2 m.

Height of balloon from girl's eye level (Perpendicular) = $88.2 - 1.2 = 87$ m.

Let balloon move from point A to B. Let eye level be at P. Q and R are points vertically below A and B on the horizontal line through P. (AQ = BR = 87 m).

Given: $\angle APQ = 60^\circ$ and $\angle BPR = 30^\circ$.

In right $\triangle APQ$:

$$\tan 60^\circ = \frac{AQ}{PQ} \Rightarrow \sqrt{3} = \frac{87}{PQ} \Rightarrow PQ = \frac{87}{\sqrt{3}} = 29\sqrt{3}\text{ m}$$

In right $\triangle BPR$:

$$\tan 30^\circ = \frac{BR}{PR} \Rightarrow \frac{1}{\sqrt{3}} = \frac{87}{PR} \Rightarrow PR = 87\sqrt{3}\text{ m}$$

Distance travelled by balloon = $PR - PQ = 87\sqrt{3} - 29\sqrt{3} = 58\sqrt{3}\text{ m}$

Hence, the distance travelled by the balloon is $58\sqrt{3}$ m.

Q 15. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.

Let tower AB = h. Car moves from point C to D.

From top A, depression angle to C is 30° ($\angle ACB = 30^\circ$) and after 6 seconds to D is 60° ($\angle ADB = 60^\circ$).

In right $\triangle ABD$:

$\tan 60^\circ = \frac{AB}{BD} \Rightarrow \sqrt{3} = \frac{h}{BD} \Rightarrow h = BD\sqrt{3}$ --- (1)

In right $\triangle ABC$:

$\tan 30^\circ = \frac{AB}{BC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{BD+CD} \Rightarrow h = \frac{BD+CD}{\sqrt{3}}$ --- (2)

From (1) and (2):

$BD\sqrt{3} = \frac{BD+CD}{\sqrt{3}} \Rightarrow 3BD = BD + CD \Rightarrow 2BD = CD \Rightarrow BD = \frac{CD}{2}$

Since the car moves with uniform speed, time taken is proportional to distance.

Time taken to cover distance CD = 6 seconds.

Therefore, time taken to cover distance BD (which is half of CD) = $\frac{6}{2} = 3$ seconds.

Hence, the car takes 3 seconds to reach the foot of the tower.