Chapter 8: Introduction to Trigonometry

Class 10 Math Chapter 8 Solutions (English Medium)

Exercise 8.1

Q 1. **In $\triangle ABC$, right-angled at B, AB = 24 cm, BC = 7 cm. Determine:**
(i) $\sin A, \cos A$
(ii) $\sin C, \cos C$

In right $\triangle ABC$, applying Pythagoras theorem:

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = (24)^2 + (7)^2 = 576 + 49 = 625$$

$$AC = \sqrt{625} = 25 \text{ cm}$$ (Hypotenuse)

(i) For angle A: Perpendicular = BC = 7 cm, Base = AB = 24 cm, Hypotenuse = AC = 25 cm

$$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{7}{25}$$

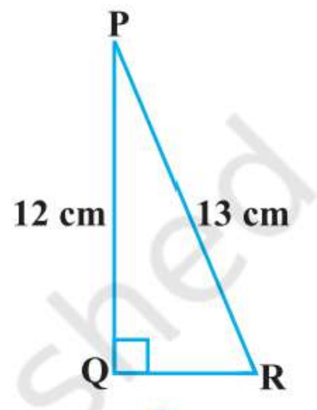
$$\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{24}{25}$$

(ii) For angle C: Perpendicular = AB = 24 cm, Base = BC = 7 cm, Hypotenuse = AC = 25 cm

$$\sin C = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{24}{25}$$

$$\cos C = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{7}{25}$$

Q 2. **In Fig. 8.13, find $\tan P - \cot R$.**



आकृति 8.13

The given figure is a right $\triangle PQR$ right-angled at Q.

Given: PQ = 12 cm and PR = 13 cm.

By Pythagoras theorem: $PR^2 = PQ^2 + QR^2$

$(13)^2 = (12)^2 + QR^2$

$169 = 144 + QR^2 \Rightarrow QR^2 = 169 - 144 = 25$

$QR = 5 \text{ cm}$

Now, for angle P: Perpendicular = QR = 5 cm, Base = PQ = 12 cm

$\tan P = \frac{\text{Perpendicular}}{\text{Base}} = \frac{QR}{PQ} = \frac{5}{12}$

For angle R: Base = QR = 5 cm, Perpendicular = PQ = 12 cm

$\cot R = \frac{\text{Base}}{\text{Perpendicular}} = \frac{QR}{PQ} = \frac{5}{12}$

Therefore, $\tan P - \cot R = \frac{5}{12} - \frac{5}{12} = 0.$

Q 3. **If $\sin A = \frac{3}{4}$, calculate $\cos A$ and $\tan A$.**

Given: $\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{3}{4}$

Let Perpendicular = 3k and Hypotenuse = 4k (where k is a positive real number).

Using Pythagoras theorem to find Base:

$$\text{Hypotenuse}^2 = \text{Perpendicular}^2 + \text{Base}^2$$

$$(4k)^2 = (3k)^2 + \text{Base}^2$$

$$16k^2 = 9k^2 + \text{Base}^2 \Rightarrow \text{Base}^2 = 7k^2 \Rightarrow \text{Base} = \sqrt{7}k$$

Now,

$$\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{\sqrt{7}k}{4k} = \frac{\sqrt{7}}{4}$$

$$\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{3k}{\sqrt{7}k} = \frac{3}{\sqrt{7}}$$

Q 4. **Given $15 \cot A = 8$, find $\sin A$ and $\sec A$.**

Given: $15 \cot A = 8 \Rightarrow \cot A = \frac{8}{15}$

We know that $\cot A = \frac{\text{Base}}{\text{Perpendicular}}$.

Let Base = 8k and Perpendicular = 15k.

Using Pythagoras theorem to find Hypotenuse:

$$\text{Hypotenuse}^2 = \text{Perpendicular}^2 + \text{Base}^2 = (15k)^2 + (8k)^2$$

$$\text{Hypotenuse}^2 = 225k^2 + 64k^2 = 289k^2 \Rightarrow \text{Hypotenuse} = 17k$$

Now,

$$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{15k}{17k} = \frac{15}{17}$$

$$\sec A = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{17k}{8k} = \frac{17}{8}$$

Q 5. Given $\sec\theta = \frac{13}{12}$, calculate all other trigonometric ratios.

Given: $\sec\theta = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{13}{12}$

Let Hypotenuse = 13k and Base = 12k.

Using Pythagoras theorem to find Perpendicular:

$$\text{Perpendicular}^2 = \text{Hypotenuse}^2 - \text{Base}^2 = (13k)^2 - (12k)^2$$

$$\text{Perpendicular}^2 = 169k^2 - 144k^2 = 25k^2 \Rightarrow \text{Perpendicular} = 5k$$

All other trigonometric ratios:

$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{5k}{13k} = \frac{5}{13}$

$\cos\theta = \frac{1}{\sec\theta} = \frac{12}{13}$

$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{5k}{12k} = \frac{5}{12}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{13}{5}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{12}{5}$

Q 6. If $\angle A$ and $\angle B$ are acute angles such that $\cos A = \cos B$, then show that $\angle A = \angle B$.

Let us consider a right-angled $\triangle ABC$ where angle C is 90°. (Here A and B are acute angles).

From the triangle:

$\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{AC}{AB}$

$\cos B = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{BC}{AB}$

Given that $\cos A = \cos B$.

Therefore, $\frac{AC}{AB} = \frac{BC}{AB} \Rightarrow AC = BC$

We know that in a triangle, angles opposite to equal sides are equal.

Since $AC = BC$, their opposite angles will be equal.

Hence, $\angle B = \angle A$ or $\angle A = \angle B$. (Proved)

Q 7. **If $\cot\theta = \frac{7}{8}$, evaluate: (i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$ (ii) $\cot^2\theta$.**

Given: $\cot\theta = \frac{7}{8} = \frac{\text{Base}}{\text{Perpendicular}}$. Let Base = 7k, Perpendicular = 8k.

Hypotenuse $= \sqrt{(8k)^2 + (7k)^2} = \sqrt{64k^2 + 49k^2} = \sqrt{113}k$

$\sin\theta = \frac{8}{\sqrt{113}}$ and $\cos\theta = \frac{7}{\sqrt{113}}$

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

Using formula $(a+b)(a-b) = a^2 - b^2$:

$= \frac{1-\sin^2\theta}{1-\cos^2\theta}$

$= \frac{1-(\frac{8}{\sqrt{113}})^2}{1-(\frac{7}{\sqrt{113}})^2} = \frac{1-\frac{64}{113}}{1-\frac{49}{113}}$

$= \frac{\frac{113-64}{113}}{\frac{113-49}{113}} = \frac{49}{64}$

(ii) $\cot^2\theta$

$\cot^2\theta = (\cot\theta)^2 = (\frac{7}{8})^2 = \frac{49}{64}$

Q 8. **If $3\cot A = 4$, check whether $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.**

Given: $3\cot A = 4 \Rightarrow \cot A = \frac{4}{3}$

$\tan A = \frac{1}{\cot A} = \frac{3}{4}$. (Here Perpendicular = 3k, Base = 4k)

Hypotenuse $= \sqrt{(3k)^2 + (4k)^2} = \sqrt{9k^2 + 16k^2} = \sqrt{25k^2} = 5k$

$\sin A = \frac{3}{5}$ and $\cos A = \frac{4}{5}$

L.H.S:

$\frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-(\frac{3}{4})^2}{1+(\frac{3}{4})^2} = \frac{1-\frac{9}{16}}{1+\frac{9}{16}} = \frac{\frac{16-9}{16}}{\frac{16+9}{16}} = \frac{7}{25}$

R.H.S:

$\cos^2 A - \sin^2 A = (\frac{4}{5})^2 - (\frac{3}{5})^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

Since L.H.S = R.H.S, **Yes, it is true.**

Q 9. **In triangle ABC, right-angled at B, if $\tan A = \frac{1}{\sqrt{3}}$, find the value of:**
(i) $\sin A \cos C + \cos A \sin C$
(ii) $\cos A \cos C - \sin A \sin C$

Given: $\tan A = \frac{1}{\sqrt{3}} = \frac{\text{Perpendicular (BC)}}{\text{Base (AB)}}$. Let BC = 1k, AB = $\sqrt{3}k$.

Hypotenuse AC by Pythagoras theorem:

$$AC = \sqrt{AB^2 + BC^2} = \sqrt{(\sqrt{3}k)^2 + (1k)^2} = \sqrt{3k^2 + 1k^2} = \sqrt{4k^2} = 2k$$

Now finding the values:

$\sin A = \frac{BC}{AC} = \frac{1}{2}, \quad \cos A = \frac{AB}{AC} = \frac{\sqrt{3}}{2}$

$\sin C = \frac{AB}{AC} = \frac{\sqrt{3}}{2}, \quad \cos C = \frac{BC}{AC} = \frac{1}{2}$

(i) $\sin A \cos C + \cos A \sin C$

$= (\frac{1}{2} \times \frac{1}{2}) + (\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}) = \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$

(ii) $\cos A \cos C - \sin A \sin C$

$= (\frac{\sqrt{3}}{2} \times \frac{1}{2}) - (\frac{1}{2} \times \frac{\sqrt{3}}{2}) = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$

Q 10. **In $\triangle PQR$, right-angled at Q, $PR + QR = 25$ cm and $PQ = 5$ cm. Determine the values of $\sin P$, $\cos P$ and $\tan P$.**

Let $QR = x$ cm. Given $PR + QR = 25 \Rightarrow PR = 25 - x$.

By Pythagoras theorem in right $\triangle PQR$: $PR^2 = PQ^2 + QR^2$

$(25 - x)^2 = (5)^2 + x^2$

$625 - 50x + x^2 = 25 + x^2$

$625 - 50x = 25 \Rightarrow 50x = 600 \Rightarrow x = 12$

So, $QR = 12$ cm (Perpendicular) and $PR = 25 - 12 = 13$ cm (Hypotenuse). Base $PQ = 5$ cm.

Now finding the values:

$\sin P = \frac{QR}{PR} = \frac{12}{13}$

$\cos P = \frac{PQ}{PR} = \frac{5}{13}$

$\tan P = \frac{QR}{PQ} = \frac{12}{5}$

Q 11. **State whether the following are true or false. Justify your answer.**

(i) The value of $\tan A$ is always less than 1.

(ii) $\sec A = \frac{12}{5}$ for some value of angle A.

(iii) $\cos A$ is the abbreviation used for the cosecant of angle A.

(iv) $\cot A$ is the product of $\cot$ and A.

(v) $\sin \theta = \frac{4}{3}$ for some angle θ.

(i) False. $\tan A = \frac{\text{Perpendicular}}{\text{Base}}$. If perpendicular > base, then $\tan A > 1$ (e.g. $\tan 60^\circ = \sqrt{3} \approx 1.732$).

(ii) True. $\sec A = \frac{\text{Hypotenuse}}{\text{Base}}$. Hypotenuse is always greater than base, and here $\frac{12}{5} > 1$, which is possible.

(iii) False. $\cos A$ is the abbreviation for 'cosine' of angle A, while 'cosecant' is abbreviated as $\csc A$.

(iv) False. $\cot A$ is a single symbol denoting the trigonometric ratio. It is not the product of 'cot' and A; 'cot' has no meaning without angle A.

(v) False. $\sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$. Since hypotenuse is the longest side, the value of $\sin \theta$ can never exceed 1 (here $\frac{4}{3} > 1$).

Exercise 8.2

Q 1. **Evaluate the following:**

(i) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

(ii) $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

(iii) $\frac{\cos 45^\circ}{\sec 30^\circ + \csc 30^\circ}$

(iv) $\frac{\sin 30^\circ + \tan 45^\circ - \csc 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$

(v) $\frac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$

(i) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

$$= \left(\frac{\sqrt{3}}{2}\right) \times \left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right)$$

$$= \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$$

(ii) $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

$$= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= 2 + \frac{3}{4} - \frac{3}{4} = 2$$

(iii) $\frac{\cos 45^\circ}{\sec 30^\circ + \csc 30^\circ}$

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + 2} = \frac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}} = \frac{\sqrt{3}}{\sqrt{2}(2+2\sqrt{3})} = \frac{\sqrt{3}}{2\sqrt{2}+2\sqrt{6}}$$

Rationalizing the denominator: $\frac{\sqrt{3}}{2\sqrt{6}+2\sqrt{2}} \times \frac{2\sqrt{6}-2\sqrt{2}}{2\sqrt{6}-2\sqrt{2}}$

$$= \frac{\sqrt{3}(2\sqrt{6}-2\sqrt{2})}{(2\sqrt{6})^2-(2\sqrt{2})^2} = \frac{2\sqrt{18}-2\sqrt{6}}{24-8} = \frac{2(3\sqrt{2}-\sqrt{6})}{16} = \frac{3\sqrt{2}-\sqrt{6}}{8}$$

(iv) $\frac{\sin 30^\circ + \tan 45^\circ - \csc 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$

$$= \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1} = \frac{\frac{3}{2} - \frac{2}{\sqrt{3}}}{\frac{3}{2} + \frac{2}{\sqrt{3}}} = \frac{\frac{3\sqrt{3}-4}{2\sqrt{3}}}{\frac{3\sqrt{3}+4}{2\sqrt{3}}}$$

$$= \frac{3\sqrt{3}-4}{3\sqrt{3}+4}$$

Rationalizing the denominator: $\frac{3\sqrt{3}-4}{3\sqrt{3}+4} \times \frac{3\sqrt{3}-4}{3\sqrt{3}-4}$

$$= \frac{(3\sqrt{3}-4)^2}{(3\sqrt{3})^2-(4)^2} = \frac{27+16-24\sqrt{3}}{27-16} = \frac{43-24\sqrt{3}}{11}$$

(v) $\frac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$

Denominator $\sin^2 30^\circ + \cos^2 30^\circ = 1$ (since $\sin^2\theta + \cos^2\theta = 1$)

Numerator $= 5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2$

$$= 5\left(\frac{1}{4}\right) + 4\left(\frac{4}{3}\right) - 1 = \frac{5}{4} + \frac{16}{3} - 1$$

$= \frac{15+64-12}{12} = \frac{79-12}{12} = \frac{67}{12}$

Q 2. **Choose the correct option and justify your choice:**

(i) $\frac{2\tan 30^\circ}{1+\tan^2 30^\circ} =$ **?**

(ii) $\frac{1-\tan^2 45^\circ}{1+\tan^2 45^\circ} =$ **?**

(iii) $\sin 2A = 2\sin A$ **is true when A =**$?$

(iv) $\frac{2\tan 30^\circ}{1-\tan^2 30^\circ} =$ **?**

(i) $\frac{2\tan 30^\circ}{1+\tan^2 30^\circ}$

$= \frac{2(1/\sqrt{3})}{1+(1/\sqrt{3})^2} = \frac{2/\sqrt{3}}{1+1/3} = \frac{2/\sqrt{3}}{4/3} = \frac{2}{\sqrt{3}} \times \frac{3}{4} = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$

This is the value of $\sin 60^\circ$. Therefore, option **(A)** $\sin 60^\circ$ is correct.

(ii) $\frac{1-\tan^2 45^\circ}{1+\tan^2 45^\circ}$

$= \frac{1-(1)^2}{1+(1)^2} = \frac{1-1}{1+1} = \frac{0}{2} = 0$

Therefore, option **(D) 0** is correct.

(iii) $\sin 2A = 2\sin A$

If we put A = 0°:

LHS = $\sin(2 \times 0^\circ) = \sin 0^\circ = 0$

RHS = $2\sin 0^\circ = 2(0) = 0$

Therefore, option **(A) 0°** is correct.

(iv) $\frac{2\tan 30^\circ}{1-\tan^2 30^\circ}$

$= \frac{2(1/\sqrt{3})}{1-(1/\sqrt{3})^2} = \frac{2/\sqrt{3}}{1-1/3} = \frac{2/\sqrt{3}}{2/3} = \frac{2}{\sqrt{3}} \times \frac{3}{2} = \frac{3}{\sqrt{3}} = \sqrt{3}$

This is the value of $\tan 60^\circ$. Therefore, option **(C)** $\tan 60^\circ$ is correct.

Q 3. **If $\tan(A+B)=\sqrt{3}$ and $\tan(A-B)=\frac{1}{\sqrt{3}}$; $0^\circ < A+B \leq 90^\circ, A > B$, find A and B.**

Given: $\tan(A+B)=\sqrt{3}$

We know that $\tan 60^\circ = \sqrt{3}$.

So, $A+B=60^\circ$ --- (Equation 1)

And $\tan(A-B)=\frac{1}{\sqrt{3}}$

We know that $\tan 30^\circ = \frac{1}{\sqrt{3}}$.

So, $A-B=30^\circ$ --- (Equation 2)

Adding (1) and (2):

$(A+B)+(A-B)=60^\circ+30^\circ$

$2A=90^\circ \Rightarrow A=45^\circ$

Putting the value of A in (1):

$45^\circ+B=60^\circ \Rightarrow B=60^\circ-45^\circ=15^\circ$

Therefore, A = 45° and B = 15°.

Q 4. **State whether the following are true or false. Justify your answer.**
(i) $\sin(A+B)=\sin A+\sin B$.
(ii) The value of $\sin\theta$ increases as θ increases.
(iii) The value of $\cos\theta$ increases as θ increases.
(iv) $\sin\theta=\cos\theta$ for all values of θ.
(v) $\cot A$ is not defined for $A=0^\circ$.

(i) False. Let A = 30° and B = 60°. LHS = $\sin(30+60)=\sin 90^\circ=1$. RHS = $\sin 30^\circ+\sin 60^\circ=\frac{1}{2}+\frac{\sqrt{3}}{2}=\frac{1+\sqrt{3}}{2}$. LHS $\neq$ RHS.

(ii) True. As θ increases from 0° to 90°, the value of $\sin\theta$ increases from 0 to 1.

(iii) False. As θ increases from 0° to 90°, the value of $\cos\theta$ decreases from 1 to 0.

(iv) False. This is true only for $\theta=45^\circ$, where $\sin 45^\circ=\cos 45^\circ=\frac{1}{\sqrt{2}}$. Not for all values.

(v) True. $\cot 0^\circ=\frac{\cos 0^\circ}{\sin 0^\circ}=\frac{1}{0}$, which is undefined.

Exercise 8.3

Q 1. **Express the trigonometric ratios $\sin A$, $\sec A$ and $\tan A$ in terms of $\cot A$.**

(i) $\tan A$ in terms of $\cot A$:

$\tan A = \frac{1}{\cot A}$

(ii) $\sec A$ in terms of $\cot A$:

We know: $\sec^2 A = 1 + \tan^2 A$

$\sec^2 A = 1 + \frac{1}{\cot^2 A} = \frac{\cot^2 A + 1}{\cot^2 A}$

$\sec A = \pm\sqrt{\frac{\cot^2 A + 1}{\cot^2 A}} = \frac{\sqrt{\cot^2 A + 1}}{\cot A}$

(iii) $\sin A$ in terms of $\cot A$:

We know: $\csc^2 A = 1 + \cot^2 A$

$\csc A = \pm\sqrt{1 + \cot^2 A}$

$\sin A = \frac{1}{\csc A} = \frac{1}{\sqrt{1 + \cot^2 A}}$

Q 2. **Write all the other trigonometric ratios of $\angle A$ in terms of $\sec A$.**

(i) $\cos A = \frac{1}{\sec A}$

(ii) $\sin A$: $\sin^2 A + \cos^2 A = 1 \Rightarrow \sin^2 A = 1 - \cos^2 A = 1 - \frac{1}{\sec^2 A} = \frac{\sec^2 A - 1}{\sec^2 A}$

$\sin A = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$

(iii) $\tan A$: $\sec^2 A - \tan^2 A = 1 \Rightarrow \tan^2 A = \sec^2 A - 1$

$\tan A = \sqrt{\sec^2 A - 1}$

(iv) $\csc A$: $= \frac{1}{\sin A} = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$

(v) $\cot A$: $= \frac{1}{\tan A} = \frac{1}{\sqrt{\sec^2 A - 1}}$

Q 3. **Choose the correct option. Justify your choice.**

(i) $9\sec^2 A - 9\tan^2 A =$ **?**

(ii) $(1+\tan\theta+\sec\theta)(1+\cot\theta-\csc\theta) =$ **?**

(iii) $(\sec A+\tan A)(1-\sin A) =$ **?**

(iv) $\frac{1+\tan^2 A}{1+\cot^2 A} =$ **?**

(i) $9\sec^2 A - 9\tan^2 A$

$= 9(\sec^2 A - \tan^2 A) = 9(1) = 9$. Thus, option **(B) 9** is correct.

(ii) $(1+\tan\theta+\sec\theta)(1+\cot\theta-\csc\theta)$

$= \left(1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)\left(1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)$

$= \left(\frac{\cos\theta+\sin\theta+1}{\cos\theta}\right)\left(\frac{\sin\theta+\cos\theta-1}{\sin\theta}\right)$

$= \frac{(\sin\theta+\cos\theta)^2-(1)^2}{\sin\theta\cos\theta} = \frac{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta-1}{\sin\theta\cos\theta}$

$= \frac{1+2\sin\theta\cos\theta-1}{\sin\theta\cos\theta} = \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta} = 2$. Thus, option **(C) 2** is correct.

(iii) $(\sec A+\tan A)(1-\sin A)$

$= \left(\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right)(1-\sin A) = \left(\frac{1+\sin A}{\cos A}\right)(1-\sin A)$

$= \frac{1-\sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A} = \cos A$. Thus, option **(D)** $\cos A$ is correct.

(iv) $\frac{1+\tan^2 A}{1+\cot^2 A}$

$= \frac{\sec^2 A}{\csc^2 A} = \frac{1/\cos^2 A}{1/\sin^2 A} = \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$. Thus, option **(D)** $\tan^2 A$ is correct.

Q 4. **Prove the following identities, where the angles involved are acute angles for which the expressions are defined:**

(i) $(\csc\theta-\cot\theta)^2=\frac{1-\cos\theta}{1+\cos\theta}$

(ii) $\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}=2\sec A$

(iii) $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\sec\theta\csc\theta$

(iv) $\frac{1+\sec A}{\sec A}=\frac{\sin^2 A}{1-\cos A}$

(v) $\frac{\cos A-\sin A+1}{\cos A+\sin A-1}=\csc A+\cot A$

(vi) $\sqrt{\frac{1+\sin A}{1-\sin A}}=\sec A+\tan A$

(vii) $\frac{\sin\theta-2\sin^3\theta}{2\cos^3\theta-\cos\theta}=\tan\theta$

(viii) $(\sin A+\csc A)^2+(\cos A+\sec A)^2=7+\tan^2 A+\cot^2 A$

(ix) $(\csc A-\sin A)(\sec A-\cos A)=\frac{1}{\tan A+\cot A}$

(x) $\frac{1+\tan^2 A}{1+\cot^2 A}=\left(\frac{1-\tan A}{1-\cot A}\right)^2=\tan^2 A$

(i) LHS: $(\csc\theta-\cot\theta)^2=\left(\frac{1}{\sin\theta}-\frac{\cos\theta}{\sin\theta}\right)^2=\left(\frac{1-\cos\theta}{\sin\theta}\right)^2$

$=\frac{(1-\cos\theta)^2}{\sin^2\theta}=\frac{(1-\cos\theta)^2}{1-\cos^2\theta}=\frac{(1-\cos\theta)(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}=\frac{1-\cos\theta}{1+\cos\theta}$ = RHS.

(ii) LHS: $\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}=\frac{\cos^2 A+(1+\sin A)^2}{\cos A(1+\sin A)}$

$=\frac{\cos^2 A+1+\sin^2 A+2\sin A}{\cos A(1+\sin A)}=\frac{(\sin^2 A+\cos^2 A)+1+2\sin A}{\cos A(1+\sin A)}$

$=\frac{1+1+2\sin A}{\cos A(1+\sin A)}=\frac{2+2\sin A}{\cos A(1+\sin A)}=\frac{2(1+\sin A)}{\cos A(1+\sin A)}=\frac{2}{\cos A}=2\sec A$ = RHS.

(iii) LHS: $\frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}}+\frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}=\frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}}+\frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$

$=\frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}-\frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$

$=\frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}=\frac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$

$=\frac{1+\sin\theta\cos\theta}{\sin\theta\cos\theta}=\frac{1}{\sin\theta\cos\theta}+1=\sec\theta\csc\theta+1$ = RHS.

(iv) LHS: $\frac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}}=\frac{\frac{\cos A+1}{\cos A}}{\frac{1}{\cos A}}=\cos A+1$

RHS: $\frac{\sin^2 A}{1-\cos A}=\frac{1-\cos^2 A}{1-\cos A}=\frac{(1-\cos A)(1+\cos A)}{1-\cos A}=1+\cos A$. Thus LHS = RHS.

(v) LHS: $\frac{\cos A-\sin A+1}{\cos A+\sin A-1}$. Dividing numerator and denominator by $\sin A$:

$=\frac{\cot A-1+\csc A}{\cot A+1-\csc A}=\frac{(\cot A+\csc A)-1}{\cot A-\csc A+1}$

$=\frac{(\cot A+\csc A)-(\csc^2 A-\cot^2 A)}{\cot A-\csc A+1}=\frac{(\csc A+\cot A)[1-(\csc A-\cot A)]}{\cot A-\csc A+1}$

$=\frac{(\csc A+\cot A)(\cot A-\csc A+1)}{\cot A-\csc A+1}=\csc A+\cot A$ = RHS.

(vi) LHS: $\sqrt{\frac{(1+\sin A)(1+\sin A)}{(1-\sin A)(1+\sin A)}}=\sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}}=\sqrt{\frac{(1+\sin A)^2}{\cos^2 A}}$

$=\frac{1+\sin A}{\cos A}=\frac{1}{\cos A}+\frac{\sin A}{\cos A}=\sec A+\tan A$ = RHS.

(vii) LHS: $\frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta-1)} = \frac{\sin\theta[1-2(1-\cos^2\theta)]}{\cos\theta(2\cos^2\theta-1)} = \tan\theta\frac{[1-2+2\cos^2\theta]}{(2\cos^2\theta-1)}$

$= \tan\theta\frac{(2\cos^2\theta-1)}{(2\cos^2\theta-1)} = \tan\theta$ = RHS.

(viii) LHS: $(\sin^2 A + \csc^2 A + 2\sin A\csc A) + (\cos^2 A + \sec^2 A + 2\cos A\sec A)$

$= (\sin^2 A + \cos^2 A) + \csc^2 A + \sec^2 A + 2(1) + 2(1)$

$= 1 + (1 + \cot^2 A) + (1 + \tan^2 A) + 4 = 7 + \tan^2 A + \cot^2 A$ = RHS.

(ix) LHS:

$\left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right) = \left(\frac{1-\sin^2 A}{\sin A}\right)\left(\frac{1-\cos^2 A}{\cos A}\right) = \frac{\cos^2 A}{\sin A} \times \frac{\sin^2 A}{\cos A} = \sin A\cos A$

RHS: $\frac{1}{\frac{\sin A}{\cos A}+\frac{\cos A}{\sin A}} = \frac{1}{\frac{\sin^2 A+\cos^2 A}{\sin A\cos A}} = \frac{1}{\frac{1}{\sin A\cos A}} = \sin A\cos A$. Thus LHS = RHS.

(x) $\frac{1+\tan^2 A}{1+\cot^2 A} = \frac{\sec^2 A}{\csc^2 A} = \frac{1/\cos^2 A}{1/\sin^2 A} = \tan^2 A$. (First part)

$\left(\frac{1-\tan A}{1-\cot A}\right)^2 = \left(\frac{1-\tan A}{1-1/\tan A}\right)^2 = \left(\frac{1-\tan A}{\frac{\tan A-1}{\tan A}}\right)^2 = (-\tan A)^2 = \tan^2 A$. Hence proved.