Chapter 7: Coordinate Geometry

Class 10 Math Chapter 7 Solutions (English Medium)

Exercise 7.1

Q 1. Find the distance between the following pairs of points:
(i) (2, 3), (4, 1)
(ii) (-5, 7), (-1, 3)
(iii) (a, b), (-a, -b)

(i) (2, 3) and (4, 1)

Distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Here $x_1 = 2, y_1 = 3$ and $x_2 = 4, y_2 = 1$

$d = \sqrt{(4-2)^2 + (1-3)^2} = \sqrt{2^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$ units.

(ii) (-5, 7) and (-1, 3)

$x_1 = -5, y_1 = 7$ and $x_2 = -1, y_2 = 3$

$d = \sqrt{(-1-(-5))^2 + (3-7)^2} = \sqrt{(-1+5)^2 + (-4)^2} = \sqrt{4^2 + 16} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$ units.

(iii) (a, b) and (-a, -b)

$x_1 = a, y_1 = b$ and $x_2 = -a, y_2 = -b$

$d = \sqrt{(-a-a)^2 + (-b-b)^2} = \sqrt{(-2a)^2 + (-2b)^2} = \sqrt{4a^2 + 4b^2} = \sqrt{4(a^2+b^2)} = 2\sqrt{a^2+b^2}$ units.

Q 2. Find the distance between the points (0, 0) and (36, 15). Can you now find the distance between the two towns A and B discussed in Section 7.2?

Using the distance formula:

$d = \sqrt{(36-0)^2 + (15-0)^2} = \sqrt{36^2 + 15^2}$

$d = \sqrt{1296 + 225} = \sqrt{1521} = 39$ units.

Yes, the distance between the two towns A and B discussed in Section 7.2 will be 39 km (if town A is at the origin and town B is at (36, 15)).

Q 3. **Determine if the points (1, 5), (2, 3) and (-2, -11) are collinear.**

Let the points be A(1, 5), B(2, 3) and C(-2, -11).

$$AB = \sqrt{(2-1)^2 + (3-5)^2} = \sqrt{1^2 + (-2)^2} = \sqrt{1+4} = \sqrt{5}$$

$$BC = \sqrt{(-2-2)^2 + (-11-3)^2} = \sqrt{(-4)^2 + (-14)^2} = \sqrt{16+196} = \sqrt{212} = 2\sqrt{53}$$

$$AC = \sqrt{(-2-1)^2 + (-11-5)^2} = \sqrt{(-3)^2 + (-16)^2} = \sqrt{9+256} = \sqrt{265}$$

Here $AB + BC \neq AC$. Therefore, the given points are **not** collinear.

Q 4. **Check whether (5, -2), (6, 4) and (7, -2) are the vertices of an isosceles triangle.**

Let the points be A(5, -2), B(6, 4) and C(7, -2).

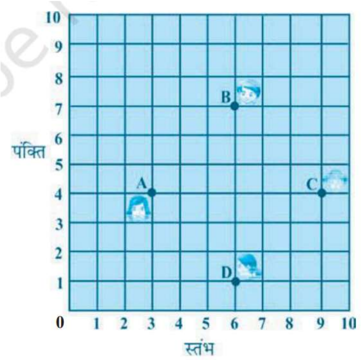
$$AB = \sqrt{(6-5)^2 + (4-(-2))^2} = \sqrt{1^2 + 6^2} = \sqrt{1+36} = \sqrt{37}$$

$$BC = \sqrt{(7-6)^2 + (-2-4)^2} = \sqrt{1^2 + (-6)^2} = \sqrt{1+36} = \sqrt{37}$$

$$AC = \sqrt{(7-5)^2 + (-2-(-2))^2} = \sqrt{2^2 + 0} = 2$$

Since $AB = BC = \sqrt{37}$, two sides are equal. Therefore, it is an **isosceles triangle**.

Q 5. **In a classroom, 4 friends are seated at the points A, B, C and D as shown in Fig. 7.8. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, "Don't you think ABCD is a square?" Chameli disagrees. Using distance formula, find which of them is correct.**



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From the figure, the coordinates are: A(3, 4), B(6, 7), C(9, 4) and D(6, 1).

Sides:

$$AB = \sqrt{(6-3)^2 + (7-4)^2} = \sqrt{3^2 + 3^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$BC = \sqrt{(9-6)^2 + (4-7)^2} = \sqrt{3^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$CD = \sqrt{(6-9)^2 + (1-4)^2} = \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$DA = \sqrt{(3-6)^2 + (4-1)^2} = \sqrt{(-3)^2 + 3^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

Diagonals:

$$AC = \sqrt{(9-3)^2 + (4-4)^2} = \sqrt{6^2 + 0} = 6$$

$$BD = \sqrt{(6-6)^2 + (1-7)^2} = \sqrt{0 + (-6)^2} = 6$$

Since all four sides are equal $(AB = BC = CD = DA)$ and the diagonals are also equal $(AC = BD)$, ABCD is a square. Therefore, **Champa is correct**.

Q 6. **Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer:**
(i) (-1, -2), (1, 0), (-1, 2), (-3, 0)
(ii) (-3, 5), (3, 1), (0, 3), (-1, -4)
(iii) (4, 5), (7, 6), (4, 3), (1, 2)

(i) A(-1, -2), B(1, 0), C(-1, 2), D(-3, 0)

$AB = \sqrt{(1+1)^2 + (0+2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$BC = \sqrt{(-1-1)^2 + (2-0)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$CD = \sqrt{(-3+1)^2 + (0-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$DA = \sqrt{(-1+3)^2 + (-2-0)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

Diagonal $AC = \sqrt{(-1+1)^2 + (2+2)^2} = \sqrt{0+16} = 4$

Diagonal $BD = \sqrt{(-3-1)^2 + (0-0)^2} = \sqrt{16+0} = 4$

All sides are equal and diagonals are equal. Hence, it is a **Square**.

(ii) A(-3, 5), B(3, 1), C(0, 3), D(-1, -4)

$AB = \sqrt{(3+3)^2 + (1-5)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$

$BC = \sqrt{(0-3)^2 + (3-1)^2} = \sqrt{9+4} = \sqrt{13}$

$AC = \sqrt{(0+3)^2 + (3-5)^2} = \sqrt{9+4} = \sqrt{13}$

Here $AC + BC = \sqrt{13} + \sqrt{13} = 2\sqrt{13} = AB$. This means A, C, and B are collinear. So, no quadrilateral is formed.

(iii) A(4, 5), B(7, 6), C(4, 3), D(1, 2)

$AB = \sqrt{(7-4)^2 + (6-5)^2} = \sqrt{9+1} = \sqrt{10}$

$BC = \sqrt{(4-7)^2 + (3-6)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

$CD = \sqrt{(1-4)^2 + (2-3)^2} = \sqrt{9+1} = \sqrt{10}$

$DA = \sqrt{(4-1)^2 + (5-2)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

Diagonal $AC = \sqrt{(4-4)^2 + (3-5)^2} = \sqrt{0+4} = 2$

Diagonal $BD = \sqrt{(1-7)^2 + (2-6)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$

Opposite sides are equal ($AB = CD, BC = DA$) but diagonals are not equal ($AC \neq BD$). Hence, it is a **Parallelogram**.

Q 7. Find the point on the x-axis which is equidistant from (2, -5) and (-2, 9).

Let the point on the x-axis be P(x, 0).

Let A(2, -5) and B(-2, 9) be the given points. Given: $PA = PB$

$\Rightarrow PA^2 = PB^2$

$(x-2)^2 + (0-(-5))^2 = (x-(-2))^2 + (0-9)^2$

$(x-2)^2 + 5^2 = (x+2)^2 + (-9)^2$

$x^2 - 4x + 4 + 25 = x^2 + 4x + 4 + 81$

$x^2 - 4x + 29 = x^2 + 4x + 85$

$-4x - 4x = 85 - 29$

$-8x = 56 \Rightarrow x = -7$

Therefore, the required point is **(-7, 0)**.

Q 8. Find the values of y for which the distance between the points P(2, -3) and Q(10, y) is 10 units.

Given: $PQ = 10 \Rightarrow PQ^2 = 100$

By distance formula: $(10-2)^2 + (y-(-3))^2 = 100$

$8^2 + (y+3)^2 = 100$

$64 + (y+3)^2 = 100$

$(y+3)^2 = 100 - 64 = 36$

Taking square root: $y + 3 = \pm 6$

If $y + 3 = 6 \Rightarrow y = 3$

If $y + 3 = -6 \Rightarrow y = -9$

Therefore, **the possible values of y are 3 or -9.**

Q 9. If Q(0, 1) is equidistant from P(5, -3) and R(x, 6), find the values of x. Also find the distances QR and PR.

Given: $PQ = QR \Rightarrow PQ^2 = QR^2$

$(0-5)^2 + (1-(-3))^2 = (x-0)^2 + (6-1)^2$

$25 + 4^2 = x^2 + 5^2$

$25 + 16 = x^2 + 25$

$x^2 = 16 \Rightarrow x = \pm 4$

Therefore, **the values of x are 4 or -4.**

When x = 4: R(4, 6)

$QR = \sqrt{(4-0)^2 + (6-1)^2} = \sqrt{16+25} = \sqrt{41}$

$PR = \sqrt{(4-5)^2 + (6-(-3))^2} = \sqrt{(-1)^2 + 9^2} = \sqrt{1+81} = \sqrt{82}$

When x = -4: R(-4, 6)

$QR = \sqrt{(-4-0)^2 + (6-1)^2} = \sqrt{16+25} = \sqrt{41}$

$PR = \sqrt{(-4-5)^2 + (6-(-3))^2} = \sqrt{(-9)^2 + 9^2} = \sqrt{81+81} = \sqrt{162} = 9\sqrt{2}$

Q 10. Find a relation between x and y such that the point (x, y) is equidistant from the point (3, 6) and (-3, 4).

Let P(x, y) be equidistant from A(3, 6) and B(-3, 4).

$PA = PB \Rightarrow PA^2 = PB^2$

$(x-3)^2 + (y-6)^2 = (x-(-3))^2 + (y-4)^2$

$(x^2 - 6x + 9) + (y^2 - 12y + 36) = (x^2 + 6x + 9) + (y^2 - 8y + 16)$

$-6x - 12y + 45 = 6x - 8y + 25$

Bringing all terms to one side: $-6x - 6x - 12y + 8y + 45 - 25 = 0$

$-12x - 4y + 20 = 0$

Dividing by -4: $3x + y - 5 = 0$

This is the required relation between x and y.

Exercise 7.2

Q 1. Find the coordinates of the point which divides the join of (-1, 7) and (4, -3) in the ratio 2:3.

Let the required point be P(x, y). The given points are $A(-1, 7)$ and $B(4, -3)$ and ratio is $m_1 : m_2 = 2 : 3$.

By Section formula:

$x = \frac{m_1x_2+m_2x_1}{m_1+m_2} = \frac{2(4)+3(-1)}{2+3} = \frac{8-3}{5} = \frac{5}{5} = 1$

$y = \frac{m_1y_2+m_2y_1}{m_1+m_2} = \frac{2(-3)+3(7)}{2+3} = \frac{-6+21}{5} = \frac{15}{5} = 3$

Therefore, the coordinates of the point are **(1, 3)**.

Q 2. Find the coordinates of the points of trisection of the line segment joining (4, -1) and (-2, -3).

Let A(4, -1) and B(-2, -3) be the given points. Let P and Q be the points of trisection of AB (i.e., $AP = PQ = QB$).

Point P divides AB in the ratio 1:2.

Coordinates of P: $x = \frac{1(-2)+2(4)}{1+2} = \frac{-2+8}{3} = \frac{6}{3} = 2$

$y = \frac{1(-3)+2(-1)}{1+2} = \frac{-3-2}{3} = \frac{-5}{3}$

So, $P(2, -5/3)$.

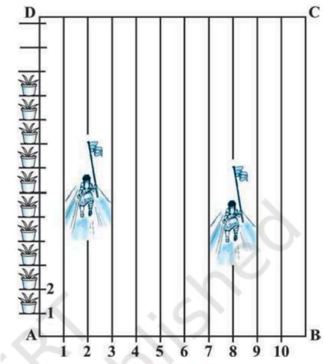
Point Q divides AB in the ratio 2:1 (or Q is the mid-point of PB).

Coordinates of Q: $x = \frac{2(-2)+1(4)}{2+1} = \frac{-4+4}{3} = 0$

$y = \frac{2(-3)+1(-1)}{2+1} = \frac{-6-1}{3} = \frac{-7}{3}$

Therefore, the coordinates of the points of trisection are **(2, -5/3)** and **(0, -7/3)**.

Q 3. **To conduct Sports Day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1m each. 100 flower pots have been placed at a distance of 1m from each other along AD, as shown in Fig. 7.12. Niharika runs 1/4 th the distance AD on the 2nd line and posts a green flag. Preet runs 1/5 th the distance AD on the 8th line and posts a red flag. What is the distance between both the flags? If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?**



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Total 100 flower pots are at a distance of 1 m on AD, so AD = 100 m.

Position of Niharika (Green flag, G): x-coordinate (line) = 2. y-coordinate = $100 \times \frac{1}{4} = 25$. Thus, **G(2, 25)**

Position of Preet (Red flag, R): x-coordinate (line) = 8. y-coordinate = $100 \times \frac{1}{5} = 20$. Thus, **R(8, 20)**

Distance between both the flags:

$GR = \sqrt{(8-2)^2 + (20-25)^2} = \sqrt{6^2 + (-5)^2} = \sqrt{36+25} = \sqrt{61}$ m.

Blue flag (B) is to be posted exactly halfway (mid-point):

Mid-point coordinates: $x = \frac{2+8}{2} = \frac{10}{2} = 5$, $y = \frac{25+20}{2} = \frac{45}{2} = 22.5$

Therefore, Rashmi should post her blue flag on the **5th line** at a distance of **22.5 m**.

Q 4. **Find the ratio in which the line segment joining the points (-3, 10) and (6, -8) is divided by (-1, 6).**

Let the point $P(-1, 6)$ divide the line segment joining $A(-3, 10)$ and $B(6, -8)$ in the ratio $k : 1$.

By Section formula for x-coordinate: $-1 = \frac{k(6)+1(-3)}{k+1}$

$-1(k+1) = 6k - 3 \Rightarrow -k - 1 = 6k - 3$

$7k = 2 \Rightarrow k = \frac{2}{7}$

Checking y-coordinate: $y = \frac{\frac{2}{7}(-8)+1(10)}{\frac{2}{7}+1} = \frac{-\frac{16}{7}+10}{\frac{9}{7}} = \frac{\frac{54}{7}}{\frac{9}{7}} = 6$ (which is correct).

Therefore, the required ratio is **2 : 7**.

Q 5. **Find the ratio in which the line segment joining A(1, -5) and B(-4, 5) is divided by the x-axis. Also find the coordinates of the point of division.**

Let the point of division on the x-axis be P(x, 0) and the ratio be $k : 1$.

Since the point lies on the x-axis, its y-coordinate is 0.

$y = \frac{k(5)+1(-5)}{k+1} = 0$

$5k - 5 = 0 \Rightarrow 5k = 5 \Rightarrow k = 1$

Therefore, the ratio is **1:1** (i.e., point P is the mid-point).

Now finding x-coordinate: $x = \frac{1(-4)+1(1)}{1+1} = \frac{-3}{2}$

Therefore, the coordinates of the point of division are **(-3/2, 0)**.

Q 6. **If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.**

Let A(1, 2), B(4, y), C(x, 6) and D(3, 5) be the vertices of the parallelogram.

We know that the diagonals of a parallelogram bisect each other. Thus, Mid-point of diagonal AC = Mid-point of diagonal BD.

Mid-point of AC: $(\frac{1+x}{2}, \frac{2+6}{2}) = (\frac{1+x}{2}, 4)$

Mid-point of BD: $(\frac{4+3}{2}, \frac{y+5}{2}) = (\frac{7}{2}, \frac{y+5}{2})$

Comparing both:

\(\frac{1+x}{2} = \frac{7}{2} \Rightarrow 1+x = 7 \Rightarrow **x = 6**\)

\(4 = \frac{y+5}{2} \Rightarrow y+5 = 8 \Rightarrow **y = 3**\)

Q 7. **Find the coordinates of a point A, where AB is the diameter of a circle whose centre is (2, -3) and B is (1, 4).**

Let the coordinates of point A be (x, y). The centre O(2, -3) will be the mid-point of the diameter AB.

By Mid-point formula: $\frac{x+1}{2} = 2 \Rightarrow x + 1 = 4 \Rightarrow x = 3$

$\frac{y+4}{2} = -3 \Rightarrow y + 4 = -6 \Rightarrow y = -10$

Therefore, the coordinates of A are **(3, -10)**.

Q 8. If A and B are (-2, -2) and (2, -4), respectively, find the coordinates of P such that $AP = \frac{3}{7}AB$ and P lies on the line segment AB.

Given: $AP = \frac{3}{7}AB$. This means $PB = AB - AP = AB - \frac{3}{7}AB = \frac{4}{7}AB$.

Thus, $AP : PB = \frac{3}{7}AB : \frac{4}{7}AB = 3 : 4$.

Now, P divides the line segment joining A(-2, -2) and B(2, -4) in the ratio 3:4.

$x = \frac{3(2)+4(-2)}{3+4} = \frac{6-8}{7} = \frac{-2}{7}$

$y = \frac{3(-4)+4(-2)}{3+4} = \frac{-12-8}{7} = \frac{-20}{7}$

Therefore, the coordinates of P are **(-2/7, -20/7)**.

Q 9. Find the coordinates of the points which divide the line segment joining A(-2, 2) and B(2, 8) into four equal parts.

Let P, Q, R divide the line segment AB into 4 equal parts. Then Q will be the mid-point of AB. P will be the mid-point of AQ, and R will be the mid-point of QB.

Coordinates of Q: $(\frac{-2+2}{2}, \frac{2+8}{2}) = (0, 5)$

Coordinates of P: (Mid-point of A and Q) $(\frac{-2+0}{2}, \frac{2+5}{2}) = (-1, 7/2)$

Coordinates of R: (Mid-point of Q and B) $(\frac{0+2}{2}, \frac{5+8}{2}) = (1, 13/2)$

Therefore, the required points are **(-1, 7/2), (0, 5) and (1, 13/2)**.

Q 10. Find the area of a rhombus if its vertices are (3, 0), (4, 5), (-1, 4) and (-2, -1) taken in order.

Area of rhombus = $\frac{1}{2} \times (\text{product of its diagonals})$

Let A(3, 0), B(4, 5), C(-1, 4) and D(-2, -1) be the vertices. The diagonals are AC and BD.

$AC = \sqrt{(-1-3)^2 + (4-0)^2} = \sqrt{(-4)^2 + 4^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$

$BD = \sqrt{(-2-4)^2 + (-1-5)^2} = \sqrt{(-6)^2 + (-6)^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$

Area = $\frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2} = \frac{1}{2} \times 24 \times 2 = 24$ sq. units.

Answer: 24 sq. units.