Chapter 6: Triangles

Class 10 Math Solutions (English Medium)

Exercise 6.1

Q 1. **Fill in the blanks using the correct word given in brackets:**

(i) All circles are _________. (congruent, similar)

Answer: similar (Because all circles have the same shape, even if their radii are different.)

(ii) All squares are _________. (similar, congruent)

Answer: similar

(iii) All _________ triangles are similar. (isosceles, equilateral)

Answer: equilateral (Because all their angles are 60°.)

(iv) Two polygons of the same number of sides are similar, if (a) their corresponding angles are _________ and (b) their corresponding sides are _________. (equal, proportional)

Answer: (a) equal, (b) proportional

Q 2. **Give two different examples of pair of: (i) similar figures (ii) non-similar figures.**

(i) Examples of similar figures:

1. Two equilateral triangles (of different side lengths).

2. Two circles (of different radii).

(ii) Examples of non-similar figures:

1. A square and a rectangle.

2. An equilateral triangle and a right-angled triangle.

Q 3. **State whether the following quadrilaterals are similar or not:**

Quadrilateral 1: Rhombus PQRS with each side 1.5 cm. Quadrilateral 2: Square ABCD with each side 3 cm.

The sides of rhombus PQRS and square ABCD are proportional (ratio = 1.5/3 = 1/2).

However, their corresponding angles are not equal. (All angles of square ABCD are 90°, whereas angles of rhombus PQRS are not 90°).

Therefore, these two quadrilaterals are **not similar**.

Exercise 6.2

Q 1. **In Fig, DE || BC. Find EC in (i) and AD in (ii).**

(i) Given: In $\triangle ABC$, DE || BC, AD = 1.5 cm, DB = 3 cm, AE = 1 cm

According to Thales Theorem (BPT), if DE || BC, then:

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{1.5}{3} = \frac{1}{EC} \Rightarrow \frac{1}{2} = \frac{1}{EC} \Rightarrow EC = 2 \text{ cm}$$

(ii) Given: In $\triangle ABC$, DE || BC, AE = 1.8 cm, EC = 5.4 cm, DB = 7.2 cm

According to Thales Theorem (BPT):

$$\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{AD}{7.2} = \frac{1.8}{5.4} \Rightarrow \frac{AD}{7.2} = \frac{1}{3}$$

$$AD = \frac{7.2}{3} = 2.4 \text{ cm}$$

Q 2. **E and F are points on the sides PQ and PR respectively of a $\triangle PQR$. For each of the following cases, state whether EF || QR:**

(i) PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2.4 cm

$\frac{PE}{EQ} = \frac{3.9}{3} = 1.3$ and $\frac{PF}{FR} = \frac{3.6}{2.4} = 1.5$

Since $\frac{PE}{EQ} \neq \frac{PF}{FR}$, by the converse of BPT, EF is **not** parallel to QR.

(ii) PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm

$\frac{PE}{EQ} = \frac{4}{4.5} = \frac{40}{45} = \frac{8}{9}$ and $\frac{PF}{FR} = \frac{8}{9}$

Since $\frac{PE}{EQ} = \frac{PF}{FR}$, by the converse of BPT, **EF || QR**.

(iii) PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.36 cm

$$EQ = PQ - PE = 1.28 - 0.18 = 1.10 \text{ cm}$$

$$FR = PR - PF = 2.56 - 0.36 = 2.20 \text{ cm}$$

$\frac{PE}{EQ} = \frac{0.18}{1.10} = \frac{18}{110} = \frac{9}{55}$

$\frac{PF}{FR} = \frac{0.36}{2.20} = \frac{36}{220} = \frac{9}{55}$

Since the ratios are equal, **EF || QR**.

Q 3. **In Fig, if LM || CB and LN || CD, prove that $\frac{AM}{AB} = \frac{AN}{AD}$.**

Proof:

In $\triangle ABC$, given LM || CB.

By BPT (Thales Theorem), $\frac{AM}{AB} = \frac{AL}{AC}$ ... (1)

Similarly, in $\triangle ADC$, given LN || CD.

By BPT, $\frac{AN}{AD} = \frac{AL}{AC}$ ... (2)

Comparing equations (1) and (2):

$\frac{AM}{AB} = \frac{AN}{AD}$ (Hence proved)

Q 4. **In Fig, DE || AC and DF || AE. Prove that $\frac{BF}{FE} = \frac{BE}{EC}$.**

Proof:

In $\triangle BCA$ (or $\triangle ABC$), DE || AC (given).

By BPT, $\frac{BD}{DA} = \frac{BE}{EC}$ ... (1)

Now, in $\triangle BEA$ (or $\triangle ABE$), DF || AE (given).

By BPT, $\frac{BD}{DA} = \frac{BF}{FE}$ ... (2)

From equations (1) and (2):

$\frac{BF}{FE} = \frac{BE}{EC}$ (Hence proved)

Q 5. In Fig, DE || OQ and DF || OR. Show that EF || QR.

> **Proof:**
>
> In $\triangle PQO$, DE || OQ (given).
>
> By BPT, $\frac{PD}{DO} = \frac{PE}{EQ}$ ... (1)
>
> In $\triangle PRO$, DF || OR (given).
>
> By BPT, $\frac{PD}{DO} = \frac{PF}{FR}$ ... (2)
>
> From equations (1) and (2), $\frac{PE}{EQ} = \frac{PF}{FR}$
>
> Now in $\triangle PQR$, sides PQ and PR are divided in the same ratio by points E and F.
>
> Therefore, by the converse of BPT, **EF || QR**.

Q 6. In Fig, A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC || QR.

> **Proof:**
>
> In $\triangle POQ$, AB || PQ (given).
>
> By BPT, $\frac{OA}{AP} = \frac{OB}{BQ}$ ... (1)
>
> In $\triangle POR$, AC || PR (given).
>
> By BPT, $\frac{OA}{AP} = \frac{OC}{CR}$ ... (2)
>
> From equations (1) and (2), $\frac{OB}{BQ} = \frac{OC}{CR}$
>
> Now in $\triangle OQR$, sides OQ and OR are divided in the same ratio.
>
> Therefore, by the converse of BPT, **BC || QR**.

Q 7. Using Theorem 6.1 (BPT), prove that a line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side.

> **Proof:**
>
> Let $\triangle ABC$ be a triangle in which D is the mid-point of side AB. Thus $AD = DB$ $\Rightarrow \frac{AD}{DB} = 1$ ... (1)
>
> A line DE passing through D is drawn parallel to BC which intersects AC at E.
>
> Since DE || BC, by BPT: $\frac{AD}{DB} = \frac{AE}{EC}$
>
> Substituting from equation (1): $1 = \frac{AE}{EC} \Rightarrow AE = EC$
>
> Therefore, point E is the mid-point of side AC, meaning the line bisects AC. (Hence proved)

Q 8. Using Theorem 6.2 (Converse of BPT), prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side.

Proof:

Let $\triangle ABC$ have points D and E as the mid-points of sides AB and AC respectively.

Thus $AD = DB \Rightarrow \frac{AD}{DB} = 1$

And $AE = EC \Rightarrow \frac{AE}{EC} = 1$

From these two: $\frac{AD}{DB} = \frac{AE}{EC}$

According to the converse of BPT, if a line divides any two sides of a triangle in the same ratio, it must be parallel to the third side.

Therefore, **DE || BC**. (Hence proved)

Q 9. ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$.

Construction: Through point O, draw a line EO parallel to AB (and DC) intersecting AD at E. (EO || AB || DC)

Proof:

In $\triangle DAB$, EO || AB. By BPT:

$\frac{DE}{EA} = \frac{DO}{OB}$ ... (1)

In $\triangle ADC$, EO || DC. By BPT:

$\frac{AE}{ED} = \frac{AO}{OC} \Rightarrow \frac{DE}{EA} = \frac{CO}{AO}$ ... (2)

From equations (1) and (2): $\frac{DO}{OB} = \frac{CO}{AO}$

Rearranging gives: $\frac{AO}{BO} = \frac{CO}{DO}$ (Hence proved)

Q 10. **The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$. Show that ABCD is a trapezium.**

Construction: Through point O, draw a line EO parallel to AB intersecting AD at E. (EO || AB)

Proof:

In $\triangle DAB$, EO || AB. By BPT:

$\frac{DE}{EA} = \frac{DO}{OB}$ ... (1)

Given: $\frac{AO}{BO} = \frac{CO}{DO} \Rightarrow \frac{DO}{OB} = \frac{CO}{AO}$ ... (2)

From equations (1) and (2): $\frac{DE}{EA} = \frac{CO}{AO}$

Now in $\triangle ADC$, sides AD and AC are divided in the same ratio by points E and O.

By converse of BPT, EO || DC.

Since EO || AB (by construction) and EO || DC (proved), therefore **AB || DC**.

Since one pair of opposite sides of the quadrilateral is parallel, ABCD is a **trapezium**.

Exercise 6.3

Q 1. State which pairs of triangles in Fig are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form:

(i) In $\triangle ABC$ and $\triangle PQR$: $\angle A = \angle P = 60^\circ$, $\angle B = \angle Q = 80^\circ$, $\angle C = \angle R = 40^\circ$

Answer: By AAA similarity criterion, $\triangle ABC \sim \triangle PQR$

(ii) In $\triangle ABC$ and $\triangle QRP$: $\frac{AB}{QR} = \frac{2}{4} = \frac{1}{2}$, $\frac{BC}{RP} = \frac{2.5}{5} = \frac{1}{2}$, $\frac{CA}{PQ} = \frac{3}{6} = \frac{1}{2}$

Answer: By SSS similarity criterion, $\triangle ABC \sim \triangle QRP$

(iii) In $\triangle LMP$ and $\triangle DEF$: $\frac{MP}{ED} = \frac{2}{4} = \frac{1}{2}$, $\frac{LP}{DF} = \frac{3}{6} = \frac{1}{2}$, $\frac{LM}{EF} = \frac{2.7}{5} \neq \frac{1}{2}$

Answer: The triangles are not similar.

(iv) In $\triangle MNL$ and $\triangle QPR$: $\angle M = \angle Q = 70^\circ$, $\frac{MN}{PQ} = \frac{2.5}{5} = \frac{1}{2}$, $\frac{ML}{QR} = \frac{5}{10} = \frac{1}{2}$

Answer: By SAS similarity criterion, $\triangle MNL \sim \triangle QPR$

(v) In $\triangle ABC$ and $\triangle DEF$: $\angle A = 80^\circ$ is given, but the sides including this angle are not proportional (since AC is not given).

Answer: The triangles are not similar. (SAS criterion is not met)

(vi) In $\triangle DEF$: $\angle F = 180^\circ - (70^\circ + 80^\circ) = 30^\circ$. In $\triangle PQR$: $\angle P = 180^\circ - (80^\circ + 30^\circ) = 70^\circ$

$\angle D = \angle P = 70^\circ$, $\angle E = \angle Q = 80^\circ$, $\angle F = \angle R = 30^\circ$

Answer: By AAA similarity criterion, $\triangle DEF \sim \triangle PQR$

Q 2. In Fig, $\triangle ODC \sim \triangle OBA$, $\angle BOC = 125^\circ$ and $\angle CDO = 70^\circ$. Find $\angle DOC$, $\angle DCO$ and $\angle OAB$.

Since DOB is a straight line: $\angle DOC + \angle COB = 180^\circ$ (linear pair)

$\angle DOC = 180^\circ - 125^\circ = 55^\circ$

In $\triangle ODC$ by angle sum property: $\angle DCO + \angle CDO + \angle DOC = 180^\circ$

$\angle DCO + 70^\circ + 55^\circ = 180^\circ \Rightarrow \angle DCO = 180^\circ - 125^\circ = 55^\circ$

It is given that $\triangle ODC \sim \triangle OBA$. Therefore, corresponding angles are equal:

$\angle OAB = \angle OCD = 55^\circ$

Answer: $\angle DOC = 55^\circ$, $\angle DCO = 55^\circ$, $\angle OAB = 55^\circ$

Q 3. Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at the point O. Using a similarity criterion for two triangles, show that $\frac{OA}{OC} = \frac{OB}{OD}$.

> In $\triangle OAB$ and $\triangle OCD$:
>
> $\angle OAB = \angle OCD$ (Alternate interior angles, as AB || DC)
>
> $\angle OBA = \angle ODC$ (Alternate interior angles)
>
> $\angle AOB = \angle COD$ (Vertically opposite angles)
>
> By AA (or AAA) similarity criterion, $\triangle OAB \sim \triangle OCD$
>
> The ratio of corresponding sides of similar triangles is equal:
>
> $\frac{OA}{OC} = \frac{OB}{OD}$ (Hence proved)

Q 4. In Fig, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$. Show that $\triangle PQS \sim \triangle TQR$.

> In $\triangle PQR$, $\angle 1 = \angle 2$ (given).
>
> Sides opposite to equal angles are equal, so $PQ = PR$ ... (1)
>
> Given ratio: $\frac{QR}{QS} = \frac{QT}{PR}$
>
> Using equation (1): $\frac{QR}{QS} = \frac{QT}{PQ}$ ... (2)
>
> Now, in $\triangle PQS$ and $\triangle TQR$:
>
> From ratio $\frac{QR}{QS} = \frac{QT}{PQ}$ (From eq 2)
>
> And in both triangles $\angle Q = \angle Q$ (Common)
>
> Therefore, by SAS similarity criterion, $\triangle PQS \sim \triangle TQR$. (Hence proved)

Q 5. S and T are points on sides PR and QR of $\triangle PQR$ such that $\angle P = \angle RTS$. Show that $\triangle RPQ \sim \triangle RTS$.

> In $\triangle RPQ$ and $\triangle RTS$:
>
> $\angle RPQ = \angle RTS$ (Given)
>
> $\angle R = \angle R$ (Common angle)
>
> Therefore, by AA similarity criterion, $\triangle RPQ \sim \triangle RTS$. (Hence proved)

Q 6. In Fig, if $\triangle ABE \cong \triangle ACD$, show that $\triangle ADE \sim \triangle ABC$.

Given that $\triangle ABE \cong \triangle ACD$ (Congruent).

Corresponding parts of congruent triangles are equal (CPCT):

$AB = AC$ ... (1)

$AE = AD$ ... (2)

Dividing (2) by (1): $\frac{AD}{AB} = \frac{AE}{AC}$ ... (3)

Now in $\triangle ADE$ and $\triangle ABC$:

$\frac{AD}{AB} = \frac{AE}{AC}$ (From eq 3)

$\angle A = \angle A$ (Common)

Therefore, by SAS similarity criterion, $\triangle ADE \sim \triangle ABC$. (Hence proved)

Q 7. In Fig, altitudes AD and CE of $\triangle ABC$ intersect each other at the point P. Show that:
(i) $\triangle AEP \sim \triangle CDP$
(ii) $\triangle ABD \sim \triangle CBE$
(iii) $\triangle AEP \sim \triangle ADB$ (iv) $\triangle PDC \sim \triangle BEC$

(i) $\triangle AEP \sim \triangle CDP$:

$\angle AEP = \angle CDP = 90^\circ$ (Because they are altitudes)

$\angle APE = \angle CPD$ (Vertically opposite angles)

By AA similarity, $\triangle AEP \sim \triangle CDP$

(ii) $\triangle ABD \sim \triangle CBE$:

$\angle ADB = \angle CEB = 90^\circ$

$\angle ABD = \angle CBE$ ($\angle B$ is common)

By AA similarity, $\triangle ABD \sim \triangle CBE$

(iii) $\triangle AEP \sim \triangle ADB$:

$\angle AEP = \angle ADB = 90^\circ$

$\angle PAE = \angle DAB$ ($\angle A$ is common)

By AA similarity, $\triangle AEP \sim \triangle ADB$

(iv) $\triangle PDC \sim \triangle BEC$:

$\angle PDC = \angle BEC = 90^\circ$

$\angle PCD = \angle BCE$ ($\angle C$ is common)

By AA similarity, $\triangle PDC \sim \triangle BEC$

Q 8. **E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that $\triangle ABE \sim \triangle CFB$.**

> In $\triangle ABE$ and $\triangle CFB$:
>
> $\angle A = \angle C$ (Opposite angles of a parallelogram are equal)
>
> Since AD || BC, then AE || BC.
>
> Thus $\angle AEB = \angle CBF$ (Alternate interior angles, for transversal EB)
>
> Therefore, by AA similarity criterion, $\triangle ABE \sim \triangle CFB$. (Hence proved)

Q 9. **In Fig, ABC and AMP are two right triangles, right angled at B and M respectively. Prove that: (i) $\triangle ABC \sim \triangle AMP$ (ii) $\frac{CA}{PA} = \frac{BC}{MP}$**

> **(i)** In $\triangle ABC$ and $\triangle AMP$:
>
> $\angle ABC = \angle AMP = 90^\circ$ (Given)
>
> $\angle A = \angle A$ (Common)
>
> By AA similarity, $\triangle ABC \sim \triangle AMP$.
>
> **(ii)** Since $\triangle ABC \sim \triangle AMP$, their corresponding sides will be proportional:
>
> $\frac{CA}{PA} = \frac{BC}{MP}$ (Hence proved)

Q 10. **CD and GH are respectively the bisectors of $\angle ACB$ and $\angle EGF$ such that D and H lie on sides AB and FE of $\triangle ABC$ and $\triangle EFG$ respectively. If $\triangle ABC \sim \triangle FEG$, show that: (i) $\frac{CD}{GH} = \frac{AC}{FG}$, (ii) $\triangle DCB \sim \triangle HGE$, (iii) $\triangle DCA \sim \triangle HGF$**

Given that $\triangle ABC \sim \triangle FEG$.

Therefore $\angle A = \angle F$, $\angle B = \angle E$, and $\angle ACB = \angle FGE$.

Since CD and GH are bisectors of $\angle ACB$ and $\angle FGE$:

$\angle ACD = \angle BCD = \frac{1}{2}\angle ACB$ and $\angle FGH = \angle EGH = \frac{1}{2}\angle FGE$

Hence $\angle ACD = \angle FGH$ and $\angle BCD = \angle EGH$.

(iii) First proving this: In $\triangle DCA$ and $\triangle HGF$:

$\angle A = \angle F$ (proved above)

$\angle ACD = \angle FGH$ (proved above)

By AA similarity, $\triangle DCA \sim \triangle HGF$.

(i) Proving this: Since $\triangle DCA \sim \triangle HGF$, their corresponding sides will be proportional:

$\frac{CD}{GH} = \frac{AC}{FG}$

(ii) Proving this: In $\triangle DCB$ and $\triangle HGE$:

$\angle B = \angle E$ (known)

$\angle BCD = \angle EGH$ (proved above)

By AA similarity, $\triangle DCB \sim \triangle HGE$.

Q 11. **In Fig, E is a point on side CB produced of an isosceles $\triangle ABC$ with $AB = AC$. If $AD \perp BC$ and $EF \perp AC$, prove that $\triangle ABD \sim \triangle ECF$.**

$\triangle ABC$ is an isosceles triangle with $AB = AC$. So $\angle ABD = \angle ECF$ (since $\angle B = \angle C$)

In $\triangle ABD$ and $\triangle ECF$:

$\angle ABD = \angle ECF$ (Proved)

$\angle ADB = \angle EFC = 90^\circ$ (Given that they are perpendiculars)

Therefore, by AA similarity, $\triangle ABD \sim \triangle ECF$. (Hence proved)

Q 12. **Sides AB and BC and median AD of a $\triangle ABC$ are respectively proportional to sides PQ and QR and median PM of $\triangle PQR$. Show that $\triangle ABC \sim \triangle PQR$.**

Given: $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$

Since AD and PM are medians, $BD = \frac{1}{2}BC$ and $QM = \frac{1}{2}QR$.

Therefore, $\frac{BC}{QR} = \frac{2BD}{2QM} = \frac{BD}{QM}$

So, $\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$

By SSS similarity, $\triangle ABD \sim \triangle PQM$. This gives $\angle B = \angle Q$.

Now in $\triangle ABC$ and $\triangle PQR$:

$\frac{AB}{PQ} = \frac{BC}{QR}$ (Given)

$\angle B = \angle Q$ (Proved)

Therefore, by SAS similarity, $\triangle ABC \sim \triangle PQR$. (Hence proved)

Q 13. **D is a point on the side BC of a $\triangle ABC$ such that $\angle ADC = \angle BAC$. Show that $CA^2 = CB \cdot CD$.**

In $\triangle ADC$ and $\triangle BAC$:

$\angle ADC = \angle BAC$ (Given)

$\angle ACD = \angle BCA$ (Common angle $\angle C$)

By AA similarity, $\triangle ADC \sim \triangle BAC$.

Corresponding sides of similar triangles are proportional:

$\frac{CA}{CB} = \frac{CD}{CA}$

Cross-multiplying: $CA \times CA = CB \times CD \Rightarrow CA^2 = CB \cdot CD$ (Hence proved)

Q 14. **Sides AB and AC and median AD of a $\triangle ABC$ are respectively proportional to sides PQ and PR and median PM of another $\triangle PQR$. Show that $\triangle ABC \sim \triangle PQR$.**

Given: $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$

Construction: Produce AD to E such that $AD = DE$ and join E to C. Similarly, produce PM to N such that $PM = MN$ and join N to R.

In quadrilateral ABEC, diagonals AE and BC bisect each other at D. So ABEC is a parallelogram.

Therefore $AC = BE$ and $AB = CE$. Similarly PQNR is a parallelogram, where $PR = QN$ and $PQ = RN$.

The given ratio becomes: $\frac{CE}{RN} = \frac{AC}{PR} = \frac{2AD}{2PM} = \frac{AE}{PN}$

By SSS similarity, $\triangle ACE \sim \triangle PRN$. This gives $\angle CAD = \angle RPM$.

Similarly, $\triangle ABE \sim \triangle PQN$ can be proved, giving $\angle BAD = \angle QPM$.

Adding them: $\angle BAD + \angle CAD = \angle QPM + \angle RPM \Rightarrow \angle A = \angle P$.

Now in $\triangle ABC$ and $\triangle PQR$: $\frac{AB}{PQ} = \frac{AC}{PR}$ and $\angle A = \angle P$.

By SAS similarity, $\triangle ABC \sim \triangle PQR$. (Hence proved)

Q 15. **A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.**

Let AB be the vertical pole and BC be its shadow. $AB = 6 \text{ m}, BC = 4 \text{ m}.$

Let PQ be the tower and QR be its shadow. $PQ = h \text{ m}, QR = 28 \text{ m}.$

At the same time, the angle of elevation of the sun is equal, so $\angle C = \angle R$.

Also, $\angle B = \angle Q = 90°$ (vertical to the ground).

By AA similarity, $\triangle ABC \sim \triangle PQR$.

So, $\frac{AB}{PQ} = \frac{BC}{QR} \Rightarrow \frac{6}{h} = \frac{4}{28} \Rightarrow \frac{6}{h} = \frac{1}{7}$

$h = 6 \times 7 = 42 \text{ m}$

Answer: The height of the tower is 42 m.

Q 16. **If AD and PM are medians of triangles ABC and PQR, respectively where $\triangle ABC \sim \triangle PQR$, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$.**

Given: $\triangle ABC \sim \triangle PQR$

So $\angle B = \angle Q$ and $\frac{AB}{PQ} = \frac{BC}{QR}$.

Since AD and PM are medians, $BC = 2BD$ and $QR = 2QM$.

$\frac{AB}{PQ} = \frac{2BD}{2QM} \Rightarrow \frac{AB}{PQ} = \frac{BD}{QM}$ ... (1)

Now in $\triangle ABD$ and $\triangle PQM$:

$\frac{AB}{PQ} = \frac{BD}{QM}$ (From eq 1)

$\angle B = \angle Q$ (Known)

By SAS similarity, $\triangle ABD \sim \triangle PQM$.

Corresponding sides of similar triangles are proportional, so: $\frac{AB}{PQ} = \frac{AD}{PM}$. (Hence proved)

Exercise 6.4

Q 1. **Let $\triangle ABC \sim \triangle DEF$ and their areas be, respectively, 64 cm^2 and 121 cm^2. If EF = 15.4 cm, find BC.**

We know that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$\frac{\text{Area}(\triangle ABC)}{\text{Area}(\triangle DEF)} = \left(\frac{BC}{EF}\right)^2$

$\frac{64}{121} = \left(\frac{BC}{15.4}\right)^2$

Taking square root on both sides: $\frac{8}{11} = \frac{BC}{15.4}$

$BC = \frac{8 \times 15.4}{11} = 8 \times 1.4 = 11.2 \text{ cm}$

Answer: $BC = 11.2 \text{ cm}$

Q 2. Diagonals of a trapezium ABCD with AB || DC intersect each other at the point O. If AB = 2CD, find the ratio of the areas of triangles AOB and COD.

In $\triangle AOB$ and $\triangle COD$:

$\angle OAB = \angle OCD$ (Alternate interior angles)

$\angle AOB = \angle COD$ (Vertically opposite angles)

By AA similarity, $\triangle AOB \sim \triangle COD$.

The ratio of areas of similar triangles: $\frac{\text{Area}(\triangle AOB)}{\text{Area}(\triangle COD)} = \left(\frac{AB}{CD}\right)^2$

Given $AB = 2CD \Rightarrow \frac{AB}{CD} = \frac{2}{1}$

$\frac{\text{Area}(\triangle AOB)}{\text{Area}(\triangle COD)} = \left(\frac{2}{1}\right)^2 = \frac{4}{1}$

Answer: The ratio of the areas is 4:1.

Q 3. In Fig, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that $\frac{\text{Area}(ABC)}{\text{Area}(DBC)} = \frac{AO}{DO}$.

Construction: Draw AP $\perp$ BC from A and DM $\perp$ BC from D.

In $\triangle APO$ and $\triangle DMO$:

$\angle APO = \angle DMO = 90^\circ$

$\angle AOP = \angle DOM$ (Vertically opposite angles)

By AA similarity, $\triangle APO \sim \triangle DMO$.

Therefore, $\frac{AP}{DM} = \frac{AO}{DO}$ ... (1)

$\frac{\text{Area}(\triangle ABC)}{\text{Area}(\triangle DBC)} = \frac{\frac{1}{2}\times BC\times AP}{\frac{1}{2}\times BC\times DM} = \frac{AP}{DM}$

Using equation (1): $\frac{\text{Area}(ABC)}{\text{Area}(DBC)} = \frac{AO}{DO}$. (Hence proved)

Q 4. If the areas of two similar triangles are equal, prove that they are congruent.

Let $\triangle ABC \sim \triangle PQR$ and $\text{Area}(\triangle ABC) = \text{Area}(\triangle PQR)$.

Therefore, $\frac{\text{Area}(\triangle ABC)}{\text{Area}(\triangle PQR)} = 1$

According to the theorem: $\frac{\text{Area}(\triangle ABC)}{\text{Area}(\triangle PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2}$

So, $\frac{AB^2}{PQ^2} = 1 \Rightarrow AB = PQ$

$\frac{BC^2}{QR^2} = 1 \Rightarrow BC = QR$

$\frac{CA^2}{RP^2} = 1 \Rightarrow CA = RP$

By SSS congruence criterion, $\triangle ABC \cong \triangle PQR$. (Hence proved)

Q 5. D, E and F are respectively the mid-points of sides AB, BC and CA of $\triangle ABC$. Find the ratio of the areas of $\triangle DEF$ and $\triangle ABC$.

By Mid-point theorem, the line segment joining the mid-points of two sides of a triangle is parallel to the third side and half of it.

So, $DF \parallel BC$ and $DF = \frac{1}{2}BC \Rightarrow \frac{DF}{BC} = \frac{1}{2}$.

Similarly, $DE \parallel AC \Rightarrow \frac{DE}{AC} = \frac{1}{2}$ and $EF \parallel AB \Rightarrow \frac{EF}{AB} = \frac{1}{2}$.

In $\triangle DEF$ and $\triangle CAB$, ratio of corresponding sides is equal $(1/2)$.

By SSS similarity, $\triangle DEF \sim \triangle CAB$.

$\frac{\text{Area}(\triangle DEF)}{\text{Area}(\triangle ABC)} = \left(\frac{DF}{BC}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

Answer: 1:4

Q 6. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Let $\triangle ABC \sim \triangle PQR$ and let AD and PM be their corresponding medians.

$\frac{\text{Area}(\triangle ABC)}{\text{Area}(\triangle PQR)} = \left(\frac{AB}{PQ}\right)^2$ ... (1)

Since $\triangle ABC \sim \triangle PQR$, $\angle B = \angle Q$ and $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{2BD}{2QM} = \frac{BD}{QM}$.

In $\triangle ABD$ and $\triangle PQM$: $\frac{AB}{PQ} = \frac{BD}{QM}$ and $\angle B = \angle Q$.

By SAS similarity, $\triangle ABD \sim \triangle PQM$. Therefore, $\frac{AB}{PQ} = \frac{AD}{PM}$.

Substituting this in equation (1): $\frac{\text{Area}(\triangle ABC)}{\text{Area}(\triangle PQR)} = \left(\frac{AD}{PM}\right)^2$. (Hence proved)

Q 7. Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.

Let the side of the square be a. Then the diagonal of the square will be $d = a\sqrt{2}$.

Side of equilateral triangle described on the side $(\triangle_1)$ = a.

Side of equilateral triangle described on the diagonal $(\triangle_2)$ = $a\sqrt{2}$.

All equilateral triangles are similar. Hence the ratio of their areas is equal to the square of the ratio of their sides.

$\frac{\text{Area}(\triangle_1)}{\text{Area}(\triangle_2)} = \frac{a^2}{(a\sqrt{2})^2} = \frac{a^2}{2a^2} = \frac{1}{2}$

Therefore, $\text{Area}(\triangle_1) = \frac{1}{2}\text{Area}(\triangle_2)$**.** (Hence proved)

Q 8. **ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Ratio of the areas of triangles ABC and BDE is: (A) 2:1 (B) 1:2 (C) 4:1 (D) 1:4**

Let the side of $\triangle ABC$ be $2x$.

D is mid-point of BC, so $BD = x$. The side of $\triangle BDE$ will be x.

Equilateral triangles are similar, so $\frac{\text{Area}(\triangle ABC)}{\text{Area}(\triangle BDE)} = \frac{(BC)^2}{(BD)^2} = \frac{(2x)^2}{x^2} = \frac{4x^2}{x^2} = \frac{4}{1}$

Answer: (C) 4:1

Q 9. **Sides of two similar triangles are in the ratio 4:9. Areas of these triangles are in the ratio: (A) 2:3 (B) 4:9 (C) 81:16 (D) 16:81**

The ratio of areas of similar triangles is equal to the square of the ratio of their corresponding sides.

Ratio = $\left(\frac{4}{9}\right)^2 = \frac{16}{81}$

Answer: (D) 16:81

Exercise 6.5

Q 1. **Sides of triangles are given below. Determine which of them are right triangles. In case of a right triangle, write the length of its hypotenuse.**

(i) 7 cm, 24 cm, 25 cm

Largest side is 25 cm. $25^2 = 625$. Sum of squares of other two sides = $7^2 + 24^2 = 49 + 576 = 625$.

Since $25^2 = 7^2 + 24^2$, it **is a right triangle** and **hypotenuse = 25 cm**.

(ii) 3 cm, 8 cm, 6 cm

Largest side is 8 cm. $8^2 = 64$. Sum of other two = $3^2 + 6^2 = 9 + 36 = 45$.

Since $64 \neq 45$, it **is not a right triangle**.

(iii) 50 cm, 80 cm, 100 cm

$100^2 = 10000$. Sum of other two = $50^2 + 80^2 = 2500 + 6400 = 8900$.

Since $10000 \neq 8900$, it **is not a right triangle**.

(iv) 13 cm, 12 cm, 5 cm

$13^2 = 169$. Sum of other two = $12^2 + 5^2 = 144 + 25 = 169$.

Since $169 = 169$, it **is a right triangle** and **hypotenuse = 13 cm**.

Q 2. **PQR is a triangle right angled at P and M is a point on QR such that $PM \perp QR$. Show that $PM^2 = QM \cdot MR$.**

Let $\angle R = x$. In $\triangle PQR$, $\angle P = 90^\circ$, so $\angle Q = 90^\circ - x$.

In $\triangle PMR$ ($\angle M = 90^\circ$): $\angle MPR = 180^\circ - (90^\circ + x) = 90^\circ - x$.

In $\triangle PMQ$ ($\angle M = 90^\circ$): $\angle MPQ = 180^\circ - (90^\circ + (90^\circ - x)) = x$.

Now, in $\triangle PMQ$ and $\triangle RMP$:

$\angle PMQ = \angle RMP = 90^\circ$

$\angle MPQ = \angle R = x$

By AA similarity, $\triangle PMQ \sim \triangle RMP$.

So, $\frac{PM}{RM} = \frac{QM}{PM}$

Cross multiplying: $PM^2 = QM \cdot MR$. (Hence proved)

Q 3. **In Fig, ABD is a triangle right angled at A and $AC \perp BD$. Show that:**
(i) $AB^2 = BC \cdot BD$
(ii) $AC^2 = BC \cdot DC$ (iii) $AD^2 = BD \cdot CD$

Since a perpendicular is drawn from the right angle to the hypotenuse, the triangles on both sides are similar to the whole triangle and to each other.

i.e., $\triangle ABC \sim \triangle DBA$, $\triangle ACD \sim \triangle BCA$, and $\triangle ACD \sim \triangle DBA$.

(i) From $\triangle ABC \sim \triangle DBA$:

$\frac{AB}{DB} = \frac{BC}{BA} \Rightarrow AB^2 = BC \cdot BD$

(ii) From $\triangle ABC \sim \triangle DAC$:

$\frac{AC}{DC} = \frac{BC}{AC} \Rightarrow AC^2 = BC \cdot DC$

(iii) From $\triangle DAC \sim \triangle DBA$:

$\frac{AD}{DB} = \frac{CD}{AD} \Rightarrow AD^2 = BD \cdot CD$

Q 4. **ABC is an isosceles triangle right angled at C. Prove that $AB^2 = 2AC^2$.**

In $\triangle ABC$, $\angle C = 90^\circ$ and $AC = BC$ (Isosceles triangle).

By Pythagoras theorem: $AB^2 = AC^2 + BC^2$

Since $BC = AC$, replacing BC^2 with AC^2:

\(AB^2 = AC^2 + AC^2 \Rightarrow **AB^2 = 2AC^2**\) (Hence proved)

Q 5. ABC is an isosceles triangle with $AC = BC$. If $AB^2 = 2AC^2$, prove that ABC is a right triangle.

Given: $AB^2 = 2AC^2$

We can write this as: $AB^2 = AC^2 + AC^2$

Since $AC = BC$, replacing one AC^2 with BC^2:

$AB^2 = AC^2 + BC^2$

This satisfies the converse of Pythagoras theorem (square of the longest side equals sum of squares of other two sides).

Therefore, AB is the hypotenuse and the angle opposite to it, $\angle C$, will be $90°$.

Hence, $\triangle ABC$ **is a right triangle.** (Hence proved)

Q 6. ABC is an equilateral triangle of side $2a$. Find each of its altitudes.

Let $\triangle ABC$ be an equilateral triangle with $AB = BC = CA = 2a$.

Draw altitude AD from A to BC ($AD \perp BC$).

In an equilateral triangle, altitude bisects the opposite side. So $BD = DC = a$.

In right $\triangle ABD$, by Pythagoras theorem:

$AB^2 = AD^2 + BD^2 \Rightarrow (2a)^2 = AD^2 + a^2$

$4a^2 = AD^2 + a^2 \Rightarrow AD^2 = 3a^2 \Rightarrow AD = a\sqrt{3}$

Answer: The length of each altitude is $a\sqrt{3}$.

Q 7. Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

Let ABCD be a rhombus whose diagonals AC and BD intersect at O.

We know diagonals of a rhombus bisect each other at right angles ($\angle AOB = 90°, OA = OC, OB = OD$).

In right $\triangle AOB$, $AB^2 = OA^2 + OB^2$ ... (1)

Similarly, $BC^2 = OB^2 + OC^2$ ... (2)

$CD^2 = OC^2 + OD^2$ ... (3)

$DA^2 = OD^2 + OA^2$ ... (4)

Adding all: $AB^2 + BC^2 + CD^2 + DA^2 = 2(OA^2 + OB^2 + OC^2 + OD^2)$

Since $OA = OC = \frac{AC}{2}$ and $OB = OD = \frac{BD}{2}$

Sum = $2\left(2 \times \frac{AC^2}{4} + 2 \times \frac{BD^2}{4}\right) = 2\left(\frac{AC^2}{2} + \frac{BD^2}{2}\right) = AC^2 + BD^2$

So, $AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$. (Hence proved)

Q 8. **In Fig, O is a point in the interior of a triangle ABC, $OD \perp BC, OE \perp AC$ and $OF \perp AB$. Show that:**
(i) $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$ (ii) $AF^2 + BD^2 + CE^2 = AW^2 + CD^2 + BF^2$

(i) Join OA, OB and OC.

In right triangles AFO, BDO and CEO, by Pythagoras theorem:

$OA^2 = AF^2 + OF^2 \Rightarrow AF^2 = OA^2 - OF^2$

$OB^2 = BD^2 + OD^2 \Rightarrow BD^2 = OB^2 - OD^2$

$OC^2 = CE^2 + OE^2 \Rightarrow CE^2 = OC^2 - OE^2$

Adding all three:

$AF^2 + BD^2 + CE^2 = OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2$ (Hence proved)

(ii) Similarly, in right triangles BFO, CDO and AEO:

$BF^2 = OB^2 - OF^2, CD^2 = OC^2 - OD^2, AE^2 = OA^2 - OE^2$

So, $AE^2 + CD^2 + BF^2 = OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2$

Comparing both parts:

$AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$ (Hence proved)

Q 9. **A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from base of the wall.**

Let AB be the wall and AC be the ladder. $\angle B = 90^\circ$.

Given: Ladder $AC = 10 \text{ m}$, height of wall $AB = 8 \text{ m}$.

In $\triangle ABC$, by Pythagoras theorem: $AC^2 = AB^2 + BC^2$

$10^2 = 8^2 + BC^2 \Rightarrow 100 = 64 + BC^2 \Rightarrow BC^2 = 36$

$BC = 6 \text{ m}$

Answer: Distance of the foot of the ladder is 6 m.

Q 10. **A guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?**

Let AB be the pole and AC be the wire. $\angle B = 90^\circ$.

Height of pole $AB = 18 \text{ m}$, length of wire $AC = 24 \text{ m}$.

By Pythagoras theorem: $AC^2 = AB^2 + BC^2$

$24^2 = 18^2 + BC^2 \Rightarrow 576 = 324 + BC^2 \Rightarrow BC^2 = 576 - 324 = 252$

$BC = \sqrt{252} = 6\sqrt{7} \text{ m}$

Answer: The stake should be driven $6\sqrt{7}$ m from the base.

Q 11. **An aeroplane leaves an airport and flies due north at a speed of 1000 km/hr. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1200 km/hr. How far apart will be the two planes after $1\frac{1}{2}$ hours?**

Distance traveled by first plane (North, OA): $1000 \times 1.5 = 1500 \text{ km}$.

Distance traveled by second plane (West, OB): $1200 \times 1.5 = 1800 \text{ km}$.

Angle between North and West is 90°. So $\triangle AOB$ is a right triangle.

By Pythagoras theorem, distance between them AB:

$AB^2 = OA^2 + OB^2 = (1500)^2 + (1800)^2 = 2250000 + 3240000 = 5490000$

$AB = \sqrt{5490000} = 300\sqrt{61} \text{ km}$

Answer: They will be $300\sqrt{61}$ km apart.

Q 12. **Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between the feet of the poles is 12 m, find the distance between their tops.**

Let AB and CD be the two poles. $AB = 6 \text{ m}$, $CD = 11 \text{ m}$, and $BD = 12 \text{ m}$.

Draw AE $\perp$ CD from A. $AE = BD = 12 \text{ m}$.

$CE = CD - ED$. Since $ED = AB = 6 \text{ m}$, $CE = 11 - 6 = 5 \text{ m}$.

In right $\triangle AEC$, by Pythagoras theorem:

$AC^2 = AE^2 + CE^2 = 12^2 + 5^2 = 144 + 25 = 169$

$AC = \sqrt{169} = 13 \text{ m}$

Answer: The distance between their tops is 13 m.

Q 13. **D and E are points on the sides CA and CB respectively of a triangle ABC right angled at C. Prove that $AE^2 + BD^2 = AB^2 + DE^2$.**

In right $\triangle ACE$ ($\angle C = 90^\circ$): $AE^2 = AC^2 + CE^2$ ... (1)

In right $\triangle BCD$ ($\angle C = 90^\circ$): $BD^2 = BC^2 + CD^2$ ... (2)

Adding (1) and (2):

$$AE^2 + BD^2 = (AC^2 + CE^2) + (BC^2 + CD^2) = (AC^2 + BC^2) + (CE^2 + CD^2)$$

We know in full $\triangle ABC$, $AC^2 + BC^2 = AB^2$.

And in smaller $\triangle DCE$, $CE^2 + CD^2 = DE^2$.

Therefore, $AE^2 + BD^2 = AB^2 + DE^2$. (Hence proved)

Q 14. **The perpendicular from A on side BC of a $\triangle ABC$ intersects BC at D such that $DB = 3CD$. Prove that $2AB^2 = 2AC^2 + BC^2$.**

Given: $DB = 3CD$. Since $BC = CD + DB$,
$BC = CD + 3CD = 4CD \Rightarrow CD = \frac{1}{4}BC$ and $DB = \frac{3}{4}BC$.

In right $\triangle ABD$: $AD^2 = AB^2 - DB^2$ ... (1)

In right $\triangle ACD$: $AD^2 = AC^2 - CD^2$ ... (2)

Equating both: $AB^2 - DB^2 = AC^2 - CD^2$

$$AB^2 = AC^2 + DB^2 - CD^2$$

Putting values of DB and CD:

$$AB^2 = AC^2 + \left(\frac{3}{4}BC\right)^2 - \left(\frac{1}{4}BC\right)^2 = AC^2 + \frac{9BC^2}{16} - \frac{BC^2}{16} = AC^2 + \frac{8BC^2}{16}$$

$$AB^2 = AC^2 + \frac{1}{2}BC^2$$

Multiplying by 2: $2AB^2 = 2AC^2 + BC^2$. (Hence proved)

Q 15. **In an equilateral triangle ABC, D is a point on side BC such that $BD = \frac{1}{3}BC$. Prove that $9AD^2 = 7AB^2$.**

Construction: Draw AE $\perp$ BC.

Since ABC is equilateral, the altitude is also a median. So $BE = EC = \frac{1}{2}BC$.

Also altitude of equilateral triangle $AE = \frac{\sqrt{3}}{2}BC$.

$DE = BE - BD = \frac{1}{2}BC - \frac{1}{3}BC = \frac{1}{6}BC$.

In right $\triangle ADE$: $AD^2 = AE^2 + DE^2$

$AD^2 = \left(\frac{\sqrt{3}}{2}BC\right)^2 + \left(\frac{1}{6}BC\right)^2 = \frac{3BC^2}{4} + \frac{BC^2}{36}$

$AD^2 = \frac{27BC^2+BC^2}{36} = \frac{28BC^2}{36} = \frac{7}{9}BC^2$

Since $BC = AB$, \(AD^2 = \frac{7}{9}AB^2 \Rightarrow **9AD^2 = 7AB^2**\). (Hence proved)

Q 16. **In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.**

Let $\triangle ABC$ be equilateral with side a ($AB = BC = CA = a$).

Let AD be the altitude. So $BD = \frac{a}{2}$.

In $\triangle ABD$ by Pythagoras theorem: $AD^2 + BD^2 = AB^2$

$AD^2 + \left(\frac{a}{2}\right)^2 = a^2 \Rightarrow AD^2 + \frac{a^2}{4} = a^2 \Rightarrow AD^2 = a^2 - \frac{a^2}{4} = \frac{3a^2}{4}$

$4AD^2 = 3a^2 \Rightarrow 4 \times (\text{Altitude})^2 = 3 \times (\text{Side})^2$. (Hence proved)

Q 17. **Tick the correct answer and justify: In $\triangle ABC$, $AB = 6\sqrt{3}$ cm, $AC = 12$ cm and $BC = 6$ cm. The angle B is: (A) 120° (B) 60° (C) 90° (D) 45°**

Find the squares of all three sides:

$AB^2 = (6\sqrt{3})^2 = 36 \times 3 = 108$

$BC^2 = 6^2 = 36$

$AC^2 = 12^2 = 144$

We can see that $AB^2 + BC^2 = 108 + 36 = 144 = AC^2$.

By converse of Pythagoras theorem, angle opposite to longest side AC, which is angle B, will be a right angle (90°).

Answer: (C) 90°