

Chapter 5: Arithmetic Progressions

Class 10 Math Solutions (English Medium)

Exercise 5.1

Q1. In which of the following situations, does the list of numbers involved make an arithmetic progression, and why?

(i) The taxi fare after each km when the fare is ₹15 for the first km and ₹8 for each additional km.

Answer: Fare for 1st km (a_1) = 15

Fare for 2nd km (a_2) = 15 + 8 = 23

Fare for 3rd km (a_3) = 23 + 8 = 31

Fare for 4th km (a_4) = 31 + 8 = 39

Difference: $a_2 - a_1 = 8$, $a_3 - a_2 = 8$, $a_4 - a_3 = 8$

Since the difference between consecutive terms is constant (8), it **forms an A.P.**

(ii) The amount of air present in a cylinder when a vacuum pump removes $\frac{1}{4}$ of the air remaining in the cylinder at a time.

Answer: Let initial amount of air = x (a_1)

Air left after 1st removal (a_2) = $x - \frac{1}{4}x = \frac{3}{4}x$

Air left after 2nd removal (a_3) = $\frac{3}{4}x - \frac{1}{4}(\frac{3}{4}x) = \frac{3}{4}x - \frac{3}{16}x = \frac{9}{16}x$

Here $a_2 - a_1 = \frac{3}{4}x - x = -\frac{1}{4}x$

And $a_3 - a_2 = \frac{9}{16}x - \frac{3}{4}x = -\frac{3}{16}x$

Since $a_2 - a_1 \neq a_3 - a_2$, it **does not form an A.P.**

(iii) The cost of digging a well after every metre of digging, when it costs ₹150 for the first metre and rises by ₹50 for each subsequent metre.

Answer: Cost for 1st metre (a_1) = 150

Cost for 2nd metre (a_2) = 150 + 50 = 200

Cost for 3rd metre (a_3) = 200 + 50 = 250

Difference: $a_2 - a_1 = 50$, $a_3 - a_2 = 50$

Since the difference is constant, it **forms an A.P.**

(iv) The amount of money in the account every year, when ₹10000 is deposited at compound interest at 8% per annum.

Answer: Initial amount = 10000

Amount after 1 year (a_1) = $10000 \times (1 + \frac{8}{100}) = 10800$

Amount after 2 years (a_2) = $10000 \times (1 + \frac{8}{100})^2 = 11664$

Amount after 3 years (a_3) = $10000 \times (1 + \frac{8}{100})^3 = 12597.12$

Here $a_2 - a_1 = 864$ and $a_3 - a_2 = 933.12$

Since $a_2 - a_1 \neq a_3 - a_2$, it **does not form an A.P.**

Q 2. Write first four terms of the A.P., when the first term a and the common difference d are given as follows:

(i) $a = 10, d = 10$

$$a_1 = 10, a_2 = 10 + 10 = 20, a_3 = 20 + 10 = 30, a_4 = 30 + 10 = 40$$

First four terms: 10, 20, 30, 40

(ii) $a = -2, d = 0$

$$a_1 = -2, a_2 = -2 + 0 = -2, a_3 = -2 + 0 = -2, a_4 = -2 + 0 = -2$$

First four terms: -2, -2, -2, -2

(iii) $a = 4, d = -3$

$$a_1 = 4, a_2 = 4 + (-3) = 1, a_3 = 1 + (-3) = -2, a_4 = -2 + (-3) = -5$$

First four terms: 4, 1, -2, -5

(iv) $a = -1, d = \frac{1}{2}$

$$a_1 = -1, a_2 = -1 + \frac{1}{2} = -\frac{1}{2}, a_3 = -\frac{1}{2} + \frac{1}{2} = 0, a_4 = 0 + \frac{1}{2} = \frac{1}{2}$$

First four terms: -1, $-\frac{1}{2}$, 0, $\frac{1}{2}$

(v) $a = -1.25, d = -0.25$

$$a_1 = -1.25$$

$$a_2 = -1.25 + (-0.25) = -1.50$$

$$a_3 = -1.50 + (-0.25) = -1.75$$

$$a_4 = -1.75 + (-0.25) = -2.00$$

First four terms: -1.25, -1.50, -1.75, -2.00

Q 3. For the following A.P.s, write the first term and the common difference:

(i) 3, 1, -1, -3, ...

First term (a) = 3, Common difference (d) = $1 - 3 = -2$

(ii) -5, -1, 3, 7, ...

First term (a) = -5, Common difference (d) = $-1 - (-5) = 4$

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, \dots$

First term (a) = $\frac{1}{3}$, Common difference (d) = $\frac{5}{3} - \frac{1}{3} = \frac{4}{3}$

(iv) 0.6, 1.7, 2.8, 3.9, ...

First term (a) = 0.6, Common difference (d) = $1.7 - 0.6 = 1.1$

Q 4. Which of the following are A.P.s? If they form an A.P., find the common difference d and write three more terms.

(i) 2, 4, 8, 16, ...

$$a_2 - a_1 = 4 - 2 = 2 \text{ and } a_3 - a_2 = 8 - 4 = 4$$

Since $a_2 - a_1 \neq a_3 - a_2$, it is **not an A.P.**

(ii) $2, \frac{5}{2}, 3, \frac{7}{2}, \dots$

$$d = \frac{5}{2} - 2 = \frac{1}{2}; a_3 - a_2 = 3 - \frac{5}{2} = \frac{1}{2}$$

Common difference is same, so it **is an A.P.** ($d = \frac{1}{2}$)

$$\text{Next three terms: } \frac{7}{2} + \frac{1}{2} = 4, 4 + \frac{1}{2} = \frac{9}{2}, \frac{9}{2} + \frac{1}{2} = 5$$

(iii) -1.2, -3.2, -5.2, -7.2, ...

$$d = -3.2 - (-1.2) = -2.0$$

It **is an A.P.** ($d = -2$)

Next three terms: -9.2, -11.2, -13.2

(iv) -10, -6, -2, 2, ...

$$d = -6 - (-10) = 4$$

It **is an A.P.** ($d = 4$)

Next three terms: 6, 10, 14

(v) $3, 3 + \sqrt{2}, 3 + 2\sqrt{2}, 3 + 3\sqrt{2}, \dots$

$$d = (3 + \sqrt{2}) - 3 = \sqrt{2}$$

It **is an A.P.** ($d = \sqrt{2}$)

Next three terms: $3 + 4\sqrt{2}, 3 + 5\sqrt{2}, 3 + 6\sqrt{2}$

(vi) 0.2, 0.22, 0.222, 0.2222, ...

$$a_2 - a_1 = 0.22 - 0.2 = 0.02 \text{ and } a_3 - a_2 = 0.002$$

It is **not an A.P.**

(vii) 0, -4, -8, -12, ...

$$d = -4 - 0 = -4$$

It **is an A.P.** ($d = -4$)

Next three terms: -16, -20, -24

(viii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \dots$

$$d = 0$$

It **is an A.P.** ($d = 0$)

Next three terms: $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}$

(ix) 1, 3, 9, 27, ...

$$a_2 - a_1 = 2, a_3 - a_2 = 6$$

It is **not an A.P.**

(x) $a, 2a, 3a, 4a, \dots$

$$d = 2a - a = a$$

It **is an A.P.** ($d = a$)

Next three terms: $5a, 6a, 7a$

(xi) a, a^2, a^3, a^4, \dots

$$a_2 - a_1 = a(a - 1), a_3 - a_2 = a^2(a - 1)$$

It is **not an A.P.**

(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \dots$

Series is: $\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}, \dots$

$$d = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$$

It **is an A.P.** ($d = \sqrt{2}$)

Next three terms: $5\sqrt{2}$ (or $\sqrt{50}$), $6\sqrt{2}$ (or $\sqrt{72}$), $7\sqrt{2}$ (or $\sqrt{98}$)

(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, \dots$

$$a_2 - a_1 = \sqrt{6} - \sqrt{3} \text{ and } a_3 - a_2 = 3 - \sqrt{6}$$

It is **not an A.P.**

(xiv) $1^2, 3^2, 5^2, 7^2, \dots$

i.e., 1, 9, 25, 49...

$$a_2 - a_1 = 8, a_3 - a_2 = 16$$

It is **not an A.P.**

(xv) $1^2, 5^2, 7^2, 73, \dots$

i.e., 1, 25, 49, 73...

$$d = 25 - 1 = 24, 49 - 25 = 24, 73 - 49 = 24$$

It is **an A.P.** ($d = 24$)

$$\text{Next three terms: } 73 + 24 = 97, 97 + 24 = 121, 121 + 24 = 145$$

Exercise 5.2

Q1. Fill in the blanks in the following table, given that a is the first term, d the common difference and a_n the n th term of the A.P.:

Part	a	d	n	a_n
(i)	7	3	8	28
(ii)	-18	2	10	0
(iii)	46	-3	18	-5
(iv)	-18.9	2.5	10	3.6
(v)	3.5	0	105	3.5

Explanation: Using the formula $a_n = a + (n - 1)d$

$$(i) a_n = 7 + (8 - 1)3 = 7 + 21 = 28$$

$$(ii) 0 = -18 + (10 - 1)d \Rightarrow 9d = 18 \Rightarrow d = 2$$

$$(iii) -5 = a + (18 - 1)(-3) \Rightarrow a - 51 = -5 \Rightarrow a = 46$$

$$(iv) 3.6 = -18.9 + (n - 1)2.5 \Rightarrow 22.5 = 2.5(n - 1) \Rightarrow n - 1 = 9 \Rightarrow n = 10$$

$$(v) a_n = 3.5 + (105 - 1)0 = 3.5$$

Q2. Choose the correct choice in the following and justify:

(i) 30th term of the A.P: 10, 7, 4, ..., is (A) 97 (B) 77 (C) -77 (D) -87

Solution: $a = 10, d = 7 - 10 = -3, n = 30$

$$a_{30} = a + 29d = 10 + 29(-3) = 10 - 87 = -77$$

Answer: (C) -77

(ii) 11th term of the A.P: $-3, -\frac{1}{2}, 2, \dots$, is (A) 28 (B) 22 (C) -38 (D) $-48\frac{1}{2}$

Solution: $a = -3, d = -\frac{1}{2} - (-3) = \frac{5}{2}, n = 11$

$$a_{11} = a + 10d = -3 + 10\left(\frac{5}{2}\right) = -3 + 25 = 22$$

Answer: (B) 22

Q 3. In the following APs, find the missing terms in the boxes:

(i) 2, [], 26

$$a = 2, a_3 = 26. a + 2d = 26 \Rightarrow 2 + 2d = 26 \Rightarrow 2d = 24 \Rightarrow d = 12$$

Second term = $a + d = 2 + 12 = 14$. Thus, the box is **14**.

(ii) [], 13, [], 3

$$a_2 = a + d = 13 \text{ and } a_4 = a + 3d = 3. \text{ Subtracting: } 2d = -10 \Rightarrow d = -5$$

$$a - 5 = 13 \Rightarrow a = 18. \text{ And } a_3 = a + 2d = 18 - 10 = 8$$

Missing terms are **18** and **8**.

(iii) 5, [], [], $9\frac{1}{2}$

$$a = 5, a_4 = \frac{19}{2}. a + 3d = \frac{19}{2} \Rightarrow 5 + 3d = \frac{19}{2} \Rightarrow 3d = \frac{9}{2} \Rightarrow d = \frac{3}{2}$$

$$\text{Second term} = 5 + \frac{3}{2} = \frac{13}{2} = 6\frac{1}{2}. \text{ Third term} = \frac{13}{2} + \frac{3}{2} = 8.$$

(iv) -4, [], [], [], [], 6

$$a = -4, a_6 = 6. a + 5d = 6 \Rightarrow -4 + 5d = 6 \Rightarrow 5d = 10 \Rightarrow d = 2$$

Terms: **-2, 0, 2, 4**

(v) [], 38, [], [], [], -22

$$a + d = 38 \text{ and } a + 5d = -22. \text{ Subtracting: } 4d = -60 \Rightarrow d = -15$$

$$a - 15 = 38 \Rightarrow a = 53.$$

$$\text{Other terms: } a_3 = 38 - 15 = 23, a_4 = 23 - 15 = 8, a_5 = 8 - 15 = -7$$

Missing terms: **53, 23, 8, -7**

Q 4. Which term of the AP: 3, 8, 13, 18, ... is 78?

Here $a = 3, d = 8 - 3 = 5$. Let n th term be 78.

$$a_n = a + (n - 1)d = 78$$

$$3 + (n - 1)5 = 78 \Rightarrow (n - 1)5 = 75 \Rightarrow n - 1 = 15 \Rightarrow n = 16$$

Answer: 16th term is 78.

Q 5. Find the number of terms in each of the following APs:

(i) 7, 13, 19, ..., 205

$$a = 7, d = 6, a_n = 205. a_n = a + (n - 1)d$$

$$205 = 7 + (n - 1)6 \Rightarrow 198 = 6(n - 1) \Rightarrow n - 1 = 33 \Rightarrow n = 34$$

Answer: 34 terms

(ii) 18, $15\frac{1}{2}$, 13, ..., -47

$$a = 18, d = \frac{31}{2} - 18 = -\frac{5}{2}, a_n = -47$$

$$-47 = 18 + (n - 1)\left(-\frac{5}{2}\right) \Rightarrow -65 = -\frac{5}{2}(n - 1) \Rightarrow 130 = 5(n - 1) \Rightarrow n - 1 = 26 \Rightarrow n = 27$$

Answer: 27 terms

Q 6. Check whether -150 is a term of the AP: 11, 8, 5, 2...

Here $a = 11, d = -3$. Let $a_n = -150$

$$-150 = 11 + (n - 1)(-3) \Rightarrow -161 = -3(n - 1) \Rightarrow n - 1 = \frac{161}{3}$$

$$n = \frac{161}{3} + 1 = \frac{164}{3} = 54.66$$

Since n is not a positive integer, **-150 is not a term of this A.P.**

Q 7. Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73.

$$\text{Given: } a_{11} = 38 \Rightarrow a + 10d = 38 \dots (1)$$

$$a_{16} = 73 \Rightarrow a + 15d = 73 \dots (2)$$

$$\text{Subtracting (1) from (2): } 5d = 35 \Rightarrow d = 7$$

$$\text{Putting } d \text{ in (1): } a + 70 = 38 \Rightarrow a = -32$$

$$\text{Now } a_{31} = a + 30d = -32 + 30(7) = -32 + 210 = 178$$

Answer: The 31st term is 178.

Q 8. An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.

Here $n = 50$, so last term $a_{50} = 106$. Given $a_3 = 12$.

$$a + 2d = 12 \dots (1)$$

$$a + 49d = 106 \dots (2)$$

$$\text{Subtracting (1) from (2): } 47d = 94 \Rightarrow d = 2$$

$$a + 2(2) = 12 \Rightarrow a = 8$$

$$\text{Now, 29th term: } a_{29} = a + 28d = 8 + 28(2) = 8 + 56 = 64$$

Answer: The 29th term is 64.

Q 9. If the 3rd and the 9th terms of an AP are 4 and -8 respectively, which term of this AP is zero?

$$a_3 = 4 \Rightarrow a + 2d = 4 \dots (1)$$

$$a_9 = -8 \Rightarrow a + 8d = -8 \dots (2)$$

$$\text{Subtracting (1) from (2): } 6d = -12 \Rightarrow d = -2$$

$$a + 2(-2) = 4 \Rightarrow a - 4 = 4 \Rightarrow a = 8$$

$$\text{Let } n\text{th term be zero: } a_n = 0 \Rightarrow 8 + (n - 1)(-2) = 0 \Rightarrow 8 = 2(n - 1) \Rightarrow n - 1 = 4 \Rightarrow n = 5$$

Answer: 5th term is zero.

Q 10. The 17th term of an AP exceeds its 10th term by 7. Find the common difference.

$$\text{Given, } a_{17} - a_{10} = 7$$

$$(a + 16d) - (a + 9d) = 7$$

$$7d = 7 \Rightarrow d = 1$$

Answer: The common difference is 1.

Q 11. Which term of the AP: 3, 15, 27, 39, ... will be 132 more than its 54th term?

Here $a = 3, d = 12$.

$$54\text{th term: } a_{54} = a + 53d = 3 + 53(12) = 3 + 636 = 639$$

$$\text{Let } n\text{th term be 132 more than the 54th term. So } a_n = 639 + 132 = 771$$

$$3 + (n - 1)12 = 771 \Rightarrow (n - 1)12 = 768 \Rightarrow n - 1 = 64 \Rightarrow n = 65$$

Answer: 65th term.

Q 12. Two APs have the same common difference. The difference between their 100th terms is 100, what is the difference between their 1000th terms?

Let the first terms of these APs be A and a respectively, and common difference be d .

$$\text{Difference between 100th terms: } (A + 99d) - (a + 99d) = 100 \Rightarrow A - a = 100$$

$$\text{Difference between 1000th terms: } (A + 999d) - (a + 999d) = A - a$$

Since $A - a = 100$, the difference between 1000th terms is also 100.

Answer: 100

Q 13. How many three-digit numbers are divisible by 7?

Smallest 3-digit number divisible by 7 = 105 (a)

Largest 3-digit number divisible by 7 = 994 (a_n)

Here $a = 105, d = 7, a_n = 994$.

$$a_n = a + (n - 1)d \Rightarrow 994 = 105 + (n - 1)7$$

$$889 = 7(n - 1) \Rightarrow n - 1 = 127 \Rightarrow n = 128$$

Answer: 128 numbers.

Q 14. How many multiples of 4 lie between 10 and 250?

First multiple of 4 after 10 is 12 (a)

Last multiple of 4 before 250 is 248 (a_n)

Here $a = 12, d = 4, a_n = 248$.

$$248 = 12 + (n - 1)4 \Rightarrow 236 = 4(n - 1) \Rightarrow n - 1 = 59 \Rightarrow n = 60$$

Answer: 60 multiples.

Q 15. For what value of n , are the n th terms of two APs: 63, 65, 67, ... and 3, 10, 17, ... equal?

For 1st A.P.: $a = 63, d = 2$. n th term = $63 + (n - 1)2 = 61 + 2n$

For 2nd A.P.: $A = 3, D = 7$. n th term = $3 + (n - 1)7 = -4 + 7n$

Given they are equal: $61 + 2n = -4 + 7n$

$$5n = 65 \Rightarrow n = 13$$

Answer: 13th terms will be equal.

Q 16. Determine the AP whose third term is 16 and the 7th term exceeds the 5th term by 12.

$$a_3 = 16 \Rightarrow a + 2d = 16 \dots (1)$$

$$a_7 - a_5 = 12 \Rightarrow (a + 6d) - (a + 4d) = 12 \Rightarrow 2d = 12 \Rightarrow d = 6$$

Putting d in (1): $a + 12 = 16 \Rightarrow a = 4$

The AP is: 4, 10, 16, 22, ...

Answer: A.P.: 4, 10, 16, 22, ...

Q 17. Find the 20th term from the last term of the AP: 3, 8, 13, ..., 253.

If we reverse the A.P.: First term $A = 253$, and new common difference $D = -d = -5$.

20th term of reversed A.P. = 20th term from end.

$$A_{20} = A + 19D = 253 + 19(-5) = 253 - 95 = 158$$

Answer: The 20th term from the last is 158.

Q 18. The sum of the 4th and 8th terms of an AP is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the AP.

$$a_4 + a_8 = 24 \Rightarrow (a + 3d) + (a + 7d) = 24 \Rightarrow 2a + 10d = 24 \Rightarrow a + 5d = 12 \dots (1)$$

$$a_6 + a_{10} = 44 \Rightarrow (a + 5d) + (a + 9d) = 44 \Rightarrow 2a + 14d = 44 \Rightarrow a + 7d = 22 \dots (2)$$

Subtracting (1) from (2): $2d = 10 \Rightarrow d = 5$

$$a + 25 = 12 \Rightarrow a = -13$$

First three terms: $-13, -13 + 5, -13 + 10 \Rightarrow -13, -8, -3$

Answer: -13, -8, -3

- Q** Subba Rao started work in 1995 at an annual salary of ₹5000 and received an increment of ₹200 each year. In 19. which year did his income reach ₹7000?

Initial salary (a) = 5000, Increment (d) = 200, Final salary (a_n) = 7000

$$a_n = a + (n - 1)d \Rightarrow 7000 = 5000 + (n - 1)200$$

$$2000 = 200(n - 1) \Rightarrow n - 1 = 10 \Rightarrow n = 11$$

In the 11th year, which is $1995 + 11 - 1 = 2005$.

Answer: In the 11th year (2005).

- Q** Ramkali saved ₹5 in the first week of a year and then increased her weekly savings by ₹1.75. If in the n th week, her 20. weekly savings become ₹20.75, find n .

Here $a = 5, d = 1.75, a_n = 20.75$

$$a_n = a + (n - 1)d \Rightarrow 20.75 = 5 + (n - 1)(1.75)$$

$$15.75 = 1.75(n - 1) \Rightarrow n - 1 = \frac{15.75}{1.75} = 9 \Rightarrow n = 10$$

Answer: $n = 10$

Exercise 5.3

- Q1.** Find the sum of the following APs:

(i) 2, 7, 12, ... , to 10 terms.

$$a = 2, d = 5, n = 10$$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$S_{10} = \frac{10}{2}[2(2) + 9(5)] = 5[4 + 45] = 5 \times 49 = 245$$

(ii) -37, -33, -29, ... , to 12 terms.

$$a = -37, d = -33 - (-37) = 4, n = 12$$

$$S_{12} = \frac{12}{2}[2(-37) + 11(4)] = 6[-74 + 44] = 6 \times (-30) = -180$$

(iii) 0.6, 1.7, 2.8, ... , to 100 terms.

$$a = 0.6, d = 1.1, n = 100$$

$$S_{100} = \frac{100}{2}[2(0.6) + 99(1.1)] = 50[1.2 + 108.9] = 50 \times 110.1 = 5505$$

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \dots$, to 11 terms.

$$a = \frac{1}{15}, d = \frac{1}{12} - \frac{1}{15} = \frac{5-4}{60} = \frac{1}{60}, n = 11$$

$$S_{11} = \frac{11}{2} \left[2\left(\frac{1}{15}\right) + 10\left(\frac{1}{60}\right) \right] = \frac{11}{2} \left[\frac{2}{15} + \frac{1}{6} \right]$$

$$= \frac{11}{2} \left[\frac{4+5}{30} \right] = \frac{11}{2} \times \frac{9}{30} = \frac{11}{2} \times \frac{3}{10} = \frac{33}{20}$$

- Q2.** Find the sums given below:

(i) $7 + 10\frac{1}{2} + 14 + \dots + 84$

$$a = 7, d = \frac{21}{2} - 7 = \frac{7}{2}, \text{last term } l = 84$$

$$a_n = a + (n - 1)d \Rightarrow 84 = 7 + (n - 1)\frac{7}{2} \Rightarrow 77 = (n - 1)\frac{7}{2} \Rightarrow n - 1 = 22 \Rightarrow n = 23$$

$$S_n = \frac{n}{2}(a + l) = \frac{23}{2}(7 + 84) = \frac{23 \times 91}{2} = \frac{2093}{2} = 1046\frac{1}{2}$$

(ii) $34 + 32 + 30 + \dots + 10$

$$a = 34, d = -2, l = 10$$

$$10 = 34 + (n - 1)(-2) \Rightarrow -24 = -2(n - 1) \Rightarrow n - 1 = 12 \Rightarrow n = 13$$

$$S_{13} = \frac{13}{2}(34 + 10) = \frac{13}{2} \times 44 = 13 \times 22 = 286$$

(iii) $(-5) + (-8) + (-11) + \dots + (-230)$

$$a = -5, d = -3, l = -230$$

$$-230 = -5 + (n - 1)(-3) \Rightarrow -225 = -3(n - 1) \Rightarrow n - 1 = 75 \Rightarrow n = 76$$

$$S_{76} = \frac{76}{2}(-5 - 230) = 38 \times (-235) = -8930$$

Q 3. In an AP:**(i) given $a = 5, d = 3, a_n = 50$, find n and S_n .**

$$a_n = a + (n - 1)d \Rightarrow 50 = 5 + (n - 1)3 \Rightarrow 45 = 3(n - 1) \Rightarrow n = 16$$

$$S_{16} = \frac{16}{2}(5 + 50) = 8 \times 55 = 440$$

(ii) given $a = 7, a_{13} = 35$, find d and S_{13} .

$$a_{13} = a + 12d \Rightarrow 35 = 7 + 12d \Rightarrow 28 = 12d \Rightarrow d = \frac{28}{12} = \frac{7}{3}$$

$$S_{13} = \frac{13}{2}(7 + 35) = \frac{13}{2} \times 42 = 13 \times 21 = 273$$

(iii) given $a_{12} = 37, d = 3$, find a and S_{12} .

$$a + 11d = 37 \Rightarrow a + 33 = 37 \Rightarrow a = 4$$

$$S_{12} = \frac{12}{2}(4 + 37) = 6 \times 41 = 246$$

(iv) given $a_3 = 15, S_{10} = 125$, find d and a_{10} .

$$a + 2d = 15 \dots (1)$$

$$S_{10} = \frac{10}{2}[2a + 9d] = 125 \Rightarrow 5(2a + 9d) = 125 \Rightarrow 2a + 9d = 25 \dots (2)$$

Multiply (1) by 2: $2a + 4d = 30$. Subtracting from (2): $5d = -5 \Rightarrow d = -1$. $a + 2(-1) = 15 \Rightarrow a = 17$

$$a_{10} = a + 9d = 17 + 9(-1) = 8$$

(v) given $d = 5, S_9 = 75$, find a and a_9 .

$$S_9 = \frac{9}{2}[2a + 8d] = 75 \Rightarrow \frac{9}{2}[2a + 40] = 75 \Rightarrow 2a + 40 = \frac{150}{9} = \frac{50}{3}$$

$$2a = \frac{50}{3} - 40 = \frac{-70}{3} \Rightarrow a = \frac{-35}{3}$$

$$a_9 = a + 8d = \frac{-35}{3} + 40 = \frac{-35+120}{3} = \frac{85}{3}$$

(vi) given $a = 2, d = 8, S_n = 90$, find n and a_n .

$$S_n = \frac{n}{2}[2(2) + (n - 1)8] = 90 \Rightarrow n(4n - 2) = 90 \Rightarrow 4n^2 - 2n - 90 = 0 \Rightarrow 2n^2 - n - 45 = 0$$

$(n - 5)(2n + 9) = 0$. Since n must be a positive integer, $n = 5$.

$$a_5 = a + 4d = 2 + 4(8) = 34$$

(vii) given $a = 8, a_n = 62, S_n = 210$, find n and d .

$$S_n = \frac{n}{2}(a + a_n) \Rightarrow 210 = \frac{n}{2}(8 + 62) \Rightarrow 210 = 35n \Rightarrow n = 6$$

$$a_6 = a + 5d = 62 \Rightarrow 8 + 5d = 62 \Rightarrow 5d = 54 \Rightarrow d = \frac{54}{5}$$

(viii) given $a_n = 4, d = 2, S_n = -14$, find n and a .

$$a_n = a + (n - 1)d = 4 \Rightarrow a = 4 - 2n + 2 = 6 - 2n$$

$$S_n = \frac{n}{2}(a + a_n) = -14 \Rightarrow \frac{n}{2}(6 - 2n + 4) = -14 \Rightarrow n(5 - n) = -14$$

$$n^2 - 5n - 14 = 0 \Rightarrow (n - 7)(n + 2) = 0 \Rightarrow n = 7$$

$$a = 6 - 2(7) = 6 - 14 = -8$$

(ix) given $a = 3, n = 8, S = 192$, find d .

$$S = \frac{8}{2}[2(3) + 7d] = 192 \Rightarrow 4[6 + 7d] = 192 \Rightarrow 6 + 7d = 48 \Rightarrow 7d = 42 \Rightarrow d = 6$$

(x) given $l = 28, S = 144$, and there are total 9 terms. Find a .

$$S = \frac{n}{2}(a + l) \Rightarrow 144 = \frac{9}{2}(a + 28) \Rightarrow 288 = 9(a + 28) \Rightarrow 32 = a + 28 \Rightarrow a = 4$$

Q 4. How many terms of the AP: 9, 17, 25, ... must be taken to give a sum of 636?

Here $a = 9, d = 8, S_n = 636$.

$$\frac{n}{2}[2a + (n - 1)d] = 636 \Rightarrow \frac{n}{2}[18 + (n - 1)8] = 636 \Rightarrow n[9 + 4n - 4] = 636$$

$$4n^2 + 5n - 636 = 0$$

$$\text{By quadratic formula: } n = \frac{-5 \pm \sqrt{25 + 10176}}{8} = \frac{-5 \pm 101}{8}$$

Since n must be a positive integer: $n = \frac{96}{8} = 12$

Answer: 12 terms.

Q 5. The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

$$a = 5, l = 45, S_n = 400$$

$$S_n = \frac{n}{2}(a + l) \Rightarrow 400 = \frac{n}{2}(5 + 45) \Rightarrow 400 = 25n \Rightarrow n = 16$$

$$a_{16} = 45 \Rightarrow 5 + 15d = 45 \Rightarrow 15d = 40 \Rightarrow d = \frac{40}{15} = \frac{8}{3}$$

Answer: $n = 16, d = 8/3$

Q 6. The first and the last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

$$a = 17, l = 350, d = 9$$

$$a_n = a + (n - 1)d \Rightarrow 350 = 17 + (n - 1)9 \Rightarrow 333 = 9(n - 1) \Rightarrow n - 1 = 37 \Rightarrow n = 38$$

$$S_n = \frac{n}{2}(a + l) = \frac{38}{2}(17 + 350) = 19 \times 367 = 6973$$

Answer: 38 terms and sum is 6973.

Q 7. Find the sum of first 22 terms of an AP in which $d = 7$ and 22nd term is 149.

$$a_{22} = 149, d = 7, n = 22$$

$$a + 21d = 149 \Rightarrow a + 21(7) = 149 \Rightarrow a + 147 = 149 \Rightarrow a = 2$$

$$S_{22} = \frac{22}{2}(a + a_{22}) = 11(2 + 149) = 11 \times 151 = 1661$$

Answer: 1661

Q 8. Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

$$a_2 = 14 \Rightarrow a + d = 14 \text{ and } a_3 = 18 \Rightarrow a + 2d = 18$$

$$\text{Subtracting gives } d = 4. a + 4 = 14 \Rightarrow a = 10$$

$$S_{51} = \frac{51}{2}[2a + 50d] = 51[a + 25d] = 51[10 + 25(4)] = 51 \times 110 = 5610$$

Answer: 5610

Q 9. If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

$$S_7 = 49 \Rightarrow \frac{7}{2}[2a + 6d] = 49 \Rightarrow 7[a + 3d] = 49 \Rightarrow a + 3d = 7 \dots (1)$$

$$S_{17} = 289 \Rightarrow \frac{17}{2}[2a + 16d] = 289 \Rightarrow 17[a + 8d] = 289 \Rightarrow a + 8d = 17 \dots (2)$$

$$\text{Subtracting (1) from (2): } 5d = 10 \Rightarrow d = 2. a + 6 = 7 \Rightarrow a = 1.$$

$$S_n = \frac{n}{2}[2(1) + (n - 1)2] = \frac{n}{2}[2 + 2n - 2] = \frac{n}{2}(2n) = n^2$$

Answer: n^2

Q 10. Show that $a_1, a_2, \dots, a_n, \dots$ form an AP where a_n is defined as below: (i) $a_n = 3 + 4n$ (ii) $a_n = 9 - 5n$. Also find the sum of the first 15 terms in each case.

(i) $a_n = 3 + 4n$

$$a_1 = 7, a_2 = 11, a_3 = 15$$

Difference is 4. It forms an AP with $a = 7, d = 4$.

$$S_{15} = \frac{15}{2}[2(7) + 14(4)] = \frac{15}{2}[14 + 56] = 15 \times 35 = 525$$

(ii) $a_n = 9 - 5n$

$$a_1 = 4, a_2 = -1, a_3 = -6$$

Difference is -5. It forms an AP with $a = 4, d = -5$.

$$S_{15} = \frac{15}{2}[2(4) + 14(-5)] = \frac{15}{2}[8 - 70] = 15 \times (-31) = -465$$

- Q 11.** If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and the n th terms.

$$S_n = 4n - n^2$$

$$\text{First term } a_1 = S_1 = 4(1) - (1)^2 = 3$$

$$\text{Sum of first two terms } S_2 = 4(2) - (2)^2 = 8 - 4 = 4$$

$$\text{Second term } a_2 = S_2 - S_1 = 4 - 3 = 1$$

$$\text{Third term } a_3 = S_3 - S_2 = [4(3) - 3^2] - 4 = 3 - 4 = -1$$

$$\text{10th term } a_{10} = S_{10} - S_9 = [4(10) - 10^2] - [4(9) - 9^2] = (40 - 100) - (36 - 81) = -60 - (-45) = -15$$

$$\text{nth term } a_n = S_n - S_{n-1} = (4n - n^2) - [4(n-1) - (n-1)^2] = (4n - n^2) - [4n - 4 - (n^2 - 2n + 1)] = 5 - 2n$$

Answer: $S_1 = 3, S_2 = 4, a_2 = 1, a_3 = -1, a_{10} = -15, a_n = 5 - 2n$

- Q 12.** Find the sum of the first 40 positive integers divisible by 6.

Positive integers divisible by 6: 6, 12, 18, ...

Here $a = 6, d = 6, n = 40$.

$$S_{40} = \frac{40}{2} [2(6) + 39(6)] = 20[12 + 234] = 20[246] = 4920$$

Answer: 4920

- Q 13.** Find the sum of the first 15 multiples of 8.

Multiples of 8: 8, 16, 24, ...

Here $a = 8, d = 8, n = 15$.

$$S_{15} = \frac{15}{2} [2(8) + 14(8)] = \frac{15}{2} [16 + 112] = \frac{15}{2} [128] = 15 \times 64 = 960$$

Answer: 960

- Q 14.** Find the sum of the odd numbers between 0 and 50.

Odd numbers: 1, 3, 5, ..., 49

Here $a = 1, d = 2, l = 49$.

$$49 = 1 + (n-1)2 \Rightarrow 48 = 2(n-1) \Rightarrow n = 25$$

$$S_{25} = \frac{25}{2} (1 + 49) = \frac{25}{2} \times 50 = 25 \times 25 = 625$$

Answer: 625

- Q 15.** A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: ₹200 for the first day, ₹250 for the second day, ₹300 for the third day, etc. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?

Penalty amounts: 200, 250, 300, ... This is an A.P. where $a = 200, d = 50$ and $n = 30$ days.

$$\text{Total penalty } S_{30} = \frac{30}{2} [2(200) + 29(50)] = 15[400 + 1450] = 15[1850] = 27750$$

Answer: ₹27750

- Q 16.** A sum of ₹700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is ₹20 less than its preceding prize, find the value of each of the prizes.

Total prizes $n = 7$, total sum $S_7 = 700$. Let first prize be a , then $d = -20$.

$$S_7 = \frac{7}{2} [2a + 6(-20)] = 700 \Rightarrow 7[a - 60] = 700 \Rightarrow a - 60 = 100 \Rightarrow a = 160$$

So the prizes are: ₹160, ₹140, ₹120, ₹100, ₹80, ₹60, ₹40

Answer: 160, 140, 120, 100, 80, 60, 40 (in Rs)

- Q** In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that **17. the number of trees, that each section of each class will plant, will be the same as the class. (e.g., a section of Class I will plant 1 tree, Class II will plant 2 trees, etc., till Class XII). There are 3 sections of each class. How many trees will be planted by the students?**

There are 3 sections in each class. So trees planted by Class 1 = $1 \times 3 = 3$

Class 2 = $2 \times 3 = 6$

Class 3 = $3 \times 3 = 9$... Class 12 = $12 \times 3 = 36$

A.P. formed: 3, 6, 9, ..., 36. Here $a = 3, d = 3, n = 12, l = 36$.

Total trees $S_{12} = \frac{12}{2}(a + l) = 6(3 + 36) = 6 \times 39 = 234$

Answer: 234 trees will be planted.

- Q** A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A, of radii **18. 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm...** What is the total length of such a spiral made up of 13 consecutive semicircles? (Take $\pi = 22/7$)

Length of a semicircle $l = \pi r$.

Radii $r_1 = 0.5, r_2 = 1.0, r_3 = 1.5, \dots$

Lengths $l_1 = 0.5\pi, l_2 = 1.0\pi, l_3 = 1.5\pi, \dots$ This is an A.P. where $a = 0.5\pi, d = 0.5\pi$.

Total length for 13 semicircles ($n = 13$): $S_{13} = \frac{13}{2}[2(0.5\pi) + 12(0.5\pi)] = \frac{13}{2}[\pi + 6\pi] = \frac{13}{2} \times 7\pi$
 $= \frac{13}{2} \times 7 \times \frac{22}{7} = 13 \times 11 = 143$ cm.

Answer: 143 cm.

- Q** **200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on. In how many rows are the 200 logs placed and how many logs are in the top row?**

A.P. is: 20, 19, 18, ... where $a = 20, d = -1$, total sum $S_n = 200$.

$S_n = \frac{n}{2}[2a + (n - 1)d] \Rightarrow 200 = \frac{n}{2}[40 + (n - 1)(-1)] \Rightarrow 400 = n[41 - n]$

$n^2 - 41n + 400 = 0 \Rightarrow (n - 16)(n - 25) = 0$

$n = 16$ or $n = 25$.

If $n = 25$, logs in 25th row $a_{25} = 20 + 24(-1) = -4$ (not possible).

So $n = 16$. Logs in 16th row $a_{16} = 20 + 15(-1) = 5$.

Answer: There are 16 rows and 5 logs in the top row.

- Q** In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato, and the other potatoes **20. are placed 3 m apart in a straight line. There are 10 potatoes in the line. A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket... what is the total distance the competitor has to run?**

Distances from bucket: 5 m, 8 m, 11 m...

Distance covered to pick and drop (multiply by 2): 10, 16, 22...

This is an A.P.: $a = 10, d = 6$, and $n = 10$.

Total distance $S_{10} = \frac{10}{2}[2(10) + 9(6)] = 5[20 + 54] = 5 \times 74 = 370$ m.

Answer: 370 metres.

Exercise 5.4 (Optional)

- Q1. Which term of the AP: 121, 117, 113, ..., is its first negative term?**

Here $a = 121, d = 117 - 121 = -4$.

Let n th term be negative, i.e., $a_n < 0$.

$121 + (n - 1)(-4) < 0 \Rightarrow 121 - 4n + 4 < 0 \Rightarrow 125 < 4n \Rightarrow n > 31.25$

The smallest integer value for n is 32.

Answer: 32nd term is the first negative term.

Q 2. The sum of the third and the seventh terms of an AP is 6 and their product is 8. Find the sum of first sixteen terms of the AP.

$$a_3 + a_7 = 6 \Rightarrow (a + 2d) + (a + 6d) = 6 \Rightarrow 2a + 8d = 6 \Rightarrow a + 4d = 3 \Rightarrow a = 3 - 4d \dots (1)$$

$$a_3 \times a_7 = 8 \Rightarrow (a + 2d)(a + 6d) = 8 \dots (2)$$

$$\text{Put } a \text{ in (2): } (3 - 4d + 2d)(3 - 4d + 6d) = 8 \Rightarrow (3 - 2d)(3 + 2d) = 8 \Rightarrow 9 - 4d^2 = 8 \Rightarrow d^2 = \frac{1}{4} \Rightarrow d = \pm \frac{1}{2}$$

$$\text{Case I: } d = \frac{1}{2} \text{ then } a = 3 - 4\left(\frac{1}{2}\right) = 1. S_{16} = \frac{16}{2} [2(1) + 15\left(\frac{1}{2}\right)] = 8[2 + 7.5] = 76$$

$$\text{Case II: } d = -\frac{1}{2} \text{ then } a = 3 - 4\left(-\frac{1}{2}\right) = 5. S_{16} = \frac{16}{2} [2(5) + 15\left(-\frac{1}{2}\right)] = 8[10 - 7.5] = 20$$

Answer: Sum will be 76 or 20.

Q 3. A ladder has rungs 25 cm apart. The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top. If the top and the bottom rungs are $2\frac{1}{2}$ m apart, what is the length of the wood required for the rungs?

Total distance = 2.5 m = 250 cm. Distance between rungs = 25 cm.

$$\text{Total number of rungs } n = \left(\frac{250}{25}\right) + 1 = 11.$$

Length of bottom rung $a = 45$, top rung $l = 25$. This forms an A.P.

$$\text{Total length of wood } S_{11} = \frac{11}{2}(a + l) = \frac{11}{2}(45 + 25) = \frac{11}{2} \times 70 = 11 \times 35 = 385 \text{ cm.}$$

Answer: 385 cm wood is required.

Q 4. The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find this value of x .

House numbers: 1, 2, 3, ..., 49 (It is an A.P. with $a = 1, d = 1$).

According to question: Sum from 1 to $x - 1$ = Sum from $(x + 1)$ to 49.

$$\text{i.e., } S_{x-1} = S_{49} - S_x$$

$$\text{Using } S_n = \frac{n(n+1)}{2}: \frac{(x-1)x}{2} = \frac{49 \times 50}{2} - \frac{x(x+1)}{2}$$

$$(x - 1)x + x(x + 1) = 2450 \Rightarrow 2x^2 = 2450 \Rightarrow x^2 = 1225 \Rightarrow x = 35$$

Answer: $x = 35$

Q 5. A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete. Each step has a rise of $\frac{1}{4}$ m and a tread of $\frac{1}{2}$ m. Calculate the total volume of concrete required to build the terrace.

$$\text{Volume of 1st step } V_1 = 50 \times \frac{1}{2} \times \frac{1}{4} = \frac{50}{8} = \frac{25}{4} \text{ m}^3.$$

$$\text{Volume of 2nd step } V_2 = 50 \times \frac{1}{2} \times \left(\frac{1}{4} + \frac{1}{4}\right) = \frac{50}{4} \text{ m}^3.$$

$$\text{Volume of 3rd step } V_3 = 50 \times \frac{1}{2} \times \frac{3}{4} = \frac{75}{4} \text{ m}^3.$$

This is an A.P.: $\frac{25}{4}, \frac{50}{4}, \frac{75}{4}, \dots$ where $a = \frac{25}{4}, d = \frac{25}{4}, n = 15$.

$$\text{Total volume } S_{15} = \frac{15}{2} \left[2\left(\frac{25}{4}\right) + 14\left(\frac{25}{4}\right) \right] = \frac{15}{2} \left[\frac{50}{4} + \frac{350}{4} \right] = \frac{15}{2} \left[\frac{400}{4} \right] = \frac{15}{2} \times 100 = 750 \text{ m}^3.$$

Answer: Total volume is 750 m³.