Chapter 2: Polynomials

NCERT Solutions & Important Q&A (English Medium)

📌 Key Concepts

- **Degree of a Polynomial:** The highest power of the variable in a polynomial is called its degree.
- **Linear Polynomial:** A polynomial of degree 1, e.g., $ax + b$. It has exactly 1 zero.
- **Quadratic Polynomial:** A polynomial of degree 2, e.g., $ax^2 + bx + c$. It can have at most 2 zeroes.
- **Cubic Polynomial:** A polynomial of degree 3, e.g., $ax^3 + bx^2 + cx + d$.

📝 Relationship between Zeroes and Coefficients

If α and β are the zeroes of the quadratic polynomial $ax^2 + bx + c$, then:

- **Sum of zeroes ($\alpha + \beta$):** $= -\frac{b}{a} = -\frac{\text{(Coefficient of x)}}{\text{(Coefficient of x}^2\text{)}}$
- **Product of zeroes ($\alpha\beta$):** $= \frac{c}{a} = \frac{\text{(Constant term)}}{\text{(Coefficient of x}^2\text{)}}$

Formula to form a quadratic polynomial: $k[x^2 - (\alpha + \beta)x + \alpha\beta]$

Exercise 2.1

Concept: *The number of zeroes of a polynomial $p(x)$ is equal to the number of points where the graph of $y = p(x)$ intersects the x-axis.*

Question 1: The graphs of $y = p(x)$ are given for some polynomials $p(x)$. Find the number of zeroes of $p(x)$, in each case.

- **(i)** The graph does not intersect the x-axis at any point. (The line is parallel to the x-axis).

  Answer: The number of zeroes is **0**.
- **(ii)** The graph intersects the x-axis at only 1 point.

  Answer: The number of zeroes is **1**.
- **(iii)** The graph intersects the x-axis at 3 points.

  Answer: The number of zeroes is **3**.
- **(iv)** The graph intersects the x-axis at 2 points (parabola shape).

  Answer: The number of zeroes is **2**.
- **(v)** The graph intersects the x-axis at 4 points.

  Answer: The number of zeroes is **4**.
- **(vi)** The graph intersects the x-axis at 1 point and touches it at 2 points (total 3 points).

  Answer: The number of zeroes is **3**.

Important Examples

Example 2: Find the zeroes of the quadratic polynomial $x^2 + 7x + 10$, and verify the relationship between the zeroes and the coefficients.

Solution:

Polynomial: $p(x) = x^2 + 7x + 10$

By splitting the middle term:

$= x^2 + 5x + 2x + 10$

$= x(x + 5) + 2(x + 5)$

$= (x + 2)(x + 5)$

To find zeroes, set $p(x) = 0$:

$x + 2 = 0 \Rightarrow x = -2$

$x + 5 = 0 \Rightarrow x = -5$

Therefore, the zeroes are $\alpha = -2$ and $\beta = -5$.

Verification:

Comparing with $ax^2 + bx + c$, we get $a = 1, b = 7, c = 10$

1. Sum of zeroes = $\alpha + \beta = -2 + (-5) = -7$

From formula = $-\frac{b}{a} = -\frac{7}{1} = -7$ (Verified)

2. Product of zeroes = $\alpha\beta = (-2) \times (-5) = 10$

From formula = $\frac{c}{a} = \frac{10}{1} = 10$ (Verified)

Example 3: Find the zeroes of the polynomial $x^2 - 3$ and verify the relationship between the zeroes and the coefficients.

Solution:

Polynomial: $p(x) = x^2 - 3$

Using identity $a^2 - b^2 = (a - b)(a + b)$, we can write it as $x^2 - (\sqrt{3})^2$.

$= (x - \sqrt{3})(x + \sqrt{3})$

For zeroes, $p(x) = 0$:

Therefore, the zeroes are $\alpha = \sqrt{3}$ and $\beta = -\sqrt{3}$.

Verification:

Here $a = 1, b = 0, c = -3$.

Sum of zeroes = $\sqrt{3} + (-\sqrt{3}) = 0$. And $-\frac{b}{a} = -\frac{0}{1} = 0$ (Verified)

Product of zeroes = $\sqrt{3} \times (-\sqrt{3}) = -3$. And $\frac{c}{a} = -\frac{3}{1} = -3$ (Verified)

Example 4: Find a quadratic polynomial, the sum and product of whose zeroes are -3 and 2, respectively.

Solution:

Given: Sum of zeroes $(\alpha + \beta) = -3$

Product of zeroes $(\alpha\beta) = 2$

Formula for quadratic polynomial: $x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - (-3)x + 2$

Answer: $x^2 + 3x + 2$

Exercise 2.2

Question 1: Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

(i) $x^2 - 2x - 8$

Solution: $x^2 - 4x + 2x - 8 = x(x-4) + 2(x-4) = (x-4)(x+2)$

Zeroes: $x = 4, x = -2$

Verification: Sum = $4 + (-2) = 2$; Formula: $-\frac{-2}{1} = 2$. Product = $4 \times (-2) = -8$; Formula: $\frac{-8}{1} = -8$.

(ii) $4s^2 - 4s + 1$

Solution: $4s^2 - 2s - 2s + 1 = 2s(2s-1) - 1(2s-1) = (2s-1)(2s-1)$

Zeroes: $s = \frac{1}{2}, s = \frac{1}{2}$

Verification: Sum = $\frac{1}{2} + \frac{1}{2} = 1$; Formula: $-\frac{-4}{4} = 1$. Product = $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$; Formula: $\frac{1}{4}$.

(iii) $6x^2 - 3 - 7x$

Solution: Arranging in order $6x^2 - 7x - 3$

$= 6x^2 - 9x + 2x - 3 = 3x(2x-3) + 1(2x-3) = (2x-3)(3x+1)$

Zeroes: $x = \frac{3}{2}, x = -\frac{1}{3}$

Verification: Sum = $\frac{3}{2} + (-\frac{1}{3}) = \frac{7}{6}$; Formula: $-\frac{-7}{6} = \frac{7}{6}$. Product = $\frac{3}{2} \times (-\frac{1}{3}) = -\frac{1}{2}$; Formula: $\frac{-3}{6} = -\frac{1}{2}$.

(iv) $4u^2 + 8u$

Solution: $4u(u+2) = 0$

Zeroes: $u = 0, u = -2$

Verification: Sum = $0 + (-2) = -2$; Formula: $-\frac{8}{4} = -2$. Product = $0 \times (-2) = 0$; Formula: $\frac{0}{4} = 0$.

(v) $t^2 - 15$

Solution: $(t - \sqrt{15})(t + \sqrt{15}) = 0$

Zeroes: $t = \sqrt{15}, t = -\sqrt{15}$

Verification: Sum = $\sqrt{15} - \sqrt{15} = 0$; Formula: $-\frac{0}{1} = 0$. Product = $\sqrt{15} \times (-\sqrt{15}) = -15$; Formula: $\frac{-15}{1} = -15$.

(vi) $3x^2 - x - 4$

Solution: $3x^2 - 4x + 3x - 4 = x(3x-4) + 1(3x-4) = (3x-4)(x+1)$

Zeroes: $x = \frac{4}{3}, x = -1$

Verification: Sum = $\frac{4}{3} - 1 = \frac{1}{3}$; Formula: $-\frac{-1}{3} = \frac{1}{3}$. Product = $\frac{4}{3} \times (-1) = -\frac{4}{3}$; Formula: $\frac{-4}{3}$.

Question 2: Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

Formula: $k[x^2 - (\text{Sum})x + (\text{Product})]$

- **(i)** $\frac{1}{4}, -1$
  Polynomial: $x^2 - (\frac{1}{4})x + (-1)$
  Multiplying by 4: $4x^2 - x - 4$
- **(ii)** $\sqrt{2}, \frac{1}{3}$
  Polynomial: $x^2 - (\sqrt{2})x + \frac{1}{3}$
  Multiplying by 3: $3x^2 - 3\sqrt{2}x + 1$
- **(iii)** $0, \sqrt{5}$
  Polynomial: $x^2 - (0)x + \sqrt{5}$
  Answer: $x^2 + \sqrt{5}$
- **(iv)** $1, 1$
  Polynomial: $x^2 - (1)x + 1$
  Answer: $x^2 - x + 1$
- **(v)** $-\frac{1}{4}, \frac{1}{4}$
  Polynomial: $x^2 - (-\frac{1}{4})x + \frac{1}{4}$
  Multiplying by 4: $4x^2 + x + 1$
- **(vi)** $4, 1$
  Polynomial: $x^2 - (4)x + 1$
  Answer: $x^2 - 4x + 1$

Previous Years' Board Questions (PYQs: 2024-2026)

Question 1: If one zero of the quadratic polynomial $kx^2 + 3x + k$ is 2, then find the value of k. [CBSE 2024, RBSE 2022]

Solution: Since 2 is a zero of $p(x) = kx^2 + 3x + k$, then $p(2) = 0$.
Putting $x = 2$:
$k(2)^2 + 3(2) + k = 0$
$4k + 6 + k = 0$
$5k + 6 = 0 \Rightarrow 5k = -6 \Rightarrow k = -\frac{6}{5}$

Question 2: Find a quadratic polynomial whose zeroes are -3 and 4. [RBSE 2025, CBSE 2023]

Solution: Here the zeroes are given, not the sum and product!

$\alpha = -3$ and $\beta = 4$

Sum of zeroes = $-3 + 4 = 1$

Product of zeroes = $-3 \times 4 = -12$

Polynomial = $x^2 - (\text{Sum})x + \text{Product}$

$= x^2 - (1)x + (-12)$

Answer: $x^2 - x - 12$

Question 3: If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - 4x + 3$, then find the value of $\alpha^2\beta + \alpha\beta^2$. [CBSE 2026, RBSE 2024]

Solution: Comparing $x^2 - 4x + 3$ with $ax^2 + bx + c$: $a = 1, b = -4, c = 3$

Sum of zeroes $(\alpha + \beta) = -\frac{b}{a} = 4$

Product of zeroes $(\alpha\beta) = \frac{c}{a} = 3$

We need to find: $\alpha^2\beta + \alpha\beta^2$

Taking $\alpha\beta$ as common:

$= \alpha\beta(\alpha + \beta)$

Now substituting the values:

$= 3 \times 4 = 12$

Answer: 12

Future Expected Questions

Question 1: If the sum of the squares of zeroes of the polynomial $x^2 - 8x + k$ is 40, find the value of k. (Important for High Scorers)

Solution: Let the zeroes be α and β.

Given: $\alpha^2 + \beta^2 = 40$

We know: $\alpha + \beta = 8$ and $\alpha\beta = k$

Using the identity: $(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$

$(8)^2 = 40 + 2(k)$

$64 = 40 + 2k$

$24 = 2k \Rightarrow k = 12$

Answer: $k = 12$

Question 2: By observing the graph, if a parabola does not touch the x-axis, how many real zeroes does it have?

Solution: Zero (0). Because zeroes only exist when the graph intersects or touches the x-axis. If it does not cut the x-axis, it has no real zeroes (it has imaginary zeroes).

Question 3: What are the zeroes of the quadratic polynomial $x^2 - 11$?

Solution: $x^2 - 11 = 0 \Rightarrow x^2 = 11 \Rightarrow x = \pm\sqrt{11}$
Zeroes: $\sqrt{11}$ and $-\sqrt{11}$

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