Chapter 12: Surface Areas and Volumes

Class 10 Math Chapter 12 Solutions (English Medium)

Exercise 12.1

Note: Unless stated otherwise, take $\pi = \frac{22}{7}$.

Q 1. **2 cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboid.**

Let the side of each cube be a cm.

Volume of a cube $= a^3 = 64 \text{ cm}^3$

$a = \sqrt[3]{64} = 4 \text{ cm}$

When two cubes are joined end to end, a cuboid is formed with:

Length $(l) = a + a = 4 + 4 = 8 \text{ cm}$

Breadth $(b) = a = 4 \text{ cm}$

Height $(h) = a = 4 \text{ cm}$

Surface area of the cuboid $= 2(lb + bh + hl)$

$= 2(8 \times 4 + 4 \times 4 + 4 \times 8)$

$= 2(32 + 16 + 32) = 2(80) = 160 \text{ cm}^2$

Hence, the surface area of the resulting cuboid is 160 cm^2.

Q 2. **A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel.**

Diameter of the hemisphere $= 14 \text{ cm}$

Radius of the hemisphere $(r) = \frac{14}{2} = 7 \text{ cm}$

Radius of the cylinder (r) will also be 7 cm.

Total height of the vessel $= 13 \text{ cm}$

Height of the cylinder $(h) =$ Total height $-$ Radius of hemisphere
$= 13 - 7 = 6 \text{ cm}$

Inner surface area of the vessel = (Curved surface area of cylinder) + (Curved surface area of hemisphere)

$= 2\pi rh + 2\pi r^2$

$= 2\pi r(h + r)$

$= 2 \times \frac{22}{7} \times 7 \times (6 + 7)$

$= 44 \times 13 = 572 \text{ cm}^2$

Hence, the inner surface area of the vessel is 572 cm^2.

Q 3. **A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.**

Radius of cone and hemisphere $(r) = 3.5 \text{ cm} = \frac{7}{2} \text{ cm}$

Total height of the toy $= 15.5 \text{ cm}$

Height of the cone $(h) =$ Total height $-$ Radius of hemisphere
$= 15.5 - 3.5 = 12 \text{ cm}$

Slant height of the cone $(l) = \sqrt{r^2 + h^2}$

$l = \sqrt{(3.5)^2 + (12)^2} = \sqrt{12.25 + 144} = \sqrt{156.25} = 12.5 \text{ cm}$

Total surface area of the toy = (Curved surface area of cone) + (Curved surface area of hemisphere)

$= \pi r l + 2\pi r^2$

$= \pi r(l + 2r)$

$= \frac{22}{7} \times 3.5 \times (12.5 + 2 \times 3.5)$

$= 22 \times 0.5 \times (12.5 + 7) = 11 \times 19.5 = 214.5 \text{ cm}^2$

Hence, the total surface area of the toy is 214.5 cm^2.

Q 4. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid.

Side of the cubical block $(a) = 7 \text{ cm}$

The greatest diameter the hemisphere can have is equal to the side of the cube.

Therefore, **greatest diameter of hemisphere = 7 cm**

Radius of hemisphere $(r) = \frac{7}{2} = 3.5 \text{ cm}$

Total surface area of the solid = (Total surface area of cube) + (Curved surface area of hemisphere) - (Base area of hemisphere)

$= 6a^2 + 2\pi r^2 - \pi r^2 = 6a^2 + \pi r^2$

$= 6(7)^2 + \frac{22}{7} \times \left(\frac{7}{2}\right)^2$

$= 6 \times 49 + \frac{22}{7} \times \frac{49}{4} = 294 + \frac{11 \times 7}{2}$

$= 294 + \frac{77}{2} = 294 + 38.5 = 332.5 \text{ cm}^2$

Hence, the surface area of the solid is 332.5 cm^2.

Q 5. **A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.**

Let the edge of the cube be l.

Diameter of hemisphere $= l$, so radius $(r) = \frac{l}{2}$

Surface area of the remaining solid = (Total surface area of cube) + (Curved surface area of hemispherical depression) - (Base area of hemisphere which is cut out)

$= 6l^2 + 2\pi r^2 - \pi r^2 = 6l^2 + \pi r^2$

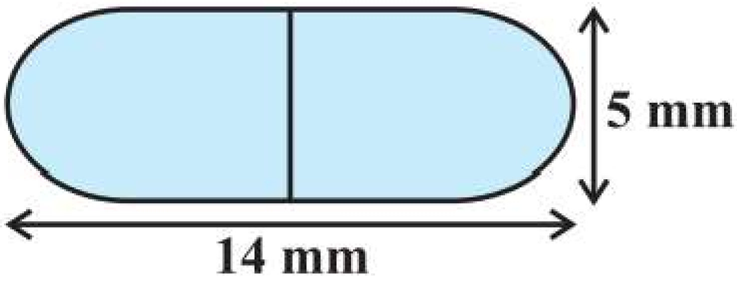
$= 6l^2 + \pi\left(\frac{l}{2}\right)^2$

$= 6l^2 + \frac{\pi l^2}{4}$

$= \frac{24l^2+\pi l^2}{4} = \frac{l^2}{4}(24 + \pi)$

Hence, the surface area of the remaining solid is $\frac{l^2}{4}(\pi + 24)$ square units.

Q 6. **A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends (see Fig. 12.10). The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area.**



आकृति 12.10

Diameter of the capsule (cylinder and hemispheres) $= 5 \text{ mm}$, so radius $(r) = \frac{5}{2} = 2.5 \text{ mm}$

Total length of the capsule $= 14 \text{ mm}$

Length of the cylindrical part (height h) = Total length - (Radii of the two hemispheres)

$h = 14 - (2.5 + 2.5) = 14 - 5 = 9 \text{ mm}$

Total surface area of capsule = (Curved surface area of cylinder) + 2 $\times$ (Curved surface area of hemisphere)

$= 2\pi rh + 2(2\pi r^2) = 2\pi rh + 4\pi r^2$

$= 2\pi r(h + 2r)$

$= 2 \times \frac{22}{7} \times 2.5 \times (9 + 2(2.5))$

$= \frac{44}{7} \times 2.5 \times (9 + 5) = \frac{110}{7} \times 14$

$= 110 \times 2 = 220 \text{ mm}^2$

Hence, the surface area of the capsule is 220 mm^2.

Q 7. **A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of Rs 500 per m^2. (Note that the base of the tent will not be covered with canvas.)**

Diameter of cylinder and cone $= 4\ \mathrm{m}$, so radius $(r) = 2\ \mathrm{m}$

Height of cylindrical part $(h) = 2.1\ \mathrm{m}$

Slant height of conical part $(l) = 2.8\ \mathrm{m}$

Area of canvas = (Curved surface area of cylinder) + (Curved surface area of cone)

$= 2\pi rh + \pi rl = \pi r(2h + l)$

$= \frac{22}{7} \times 2 \times (2(2.1) + 2.8)$

$= \frac{44}{7} \times (4.2 + 2.8) = \frac{44}{7} \times 7 = 44\ \mathrm{m}^2$

Cost of the canvas = Area $\times$ Rate

$= 44 \times 500 = \mathrm{Rs}\ 22,000$

Hence, the area of canvas used is 44 m^2 and the total cost is Rs 22,000.

Q 8. **From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2.**

Height of cylinder and conical cavity $(h) = 2.4 \mathrm{~cm}$

Diameter $= 1.4 \mathrm{~cm}$, so radius $(r) = 0.7 \mathrm{~cm}$

Slant height of the conical cavity $(l) = \sqrt{r^2 + h^2}$

$l = \sqrt{(0.7)^2 + (2.4)^2} = \sqrt{0.49 + 5.76} = \sqrt{6.25} = 2.5 \mathrm{~cm}$

Total surface area of the remaining solid = (Curved surface area of cylinder) + (Area of top base of cylinder) + (Curved surface area of conical cavity)

(Bottom base is hollowed out by the cone)

$= 2\pi rh + \pi r^2 + \pi rl = \pi r(2h + r + l)$

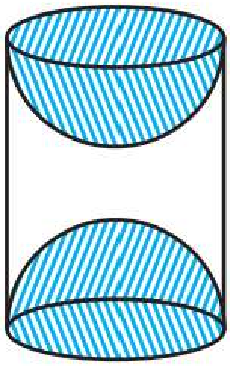
$= \frac{22}{7} \times 0.7 \times (2(2.4) + 0.7 + 2.5)$

$= 22 \times 0.1 \times (4.8 + 0.7 + 2.5) = 2.2 \times (8.0) = 17.6 \mathrm{~cm}^2$

To the nearest square centimeter: 17.6 is approximately 18.

Hence, the total surface area of the remaining solid is approximately 18 cm^2.

Q 9. **A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in Fig. 12.11. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the total surface area of the article.**



आकृति 12.11

Height of the cylinder $(h) = 10 \text{ cm}$

Radius of cylinder and hemispheres $(r) = 3.5 \text{ cm}$

Total surface area of the article = (Curved surface area of cylinder) + 2 $\times$ (Curved surface area of hemispherical scoop)

$= 2\pi rh + 2(2\pi r^2) = 2\pi rh + 4\pi r^2 = 2\pi r(h + 2r)$

$= 2 \times \frac{22}{7} \times 3.5 \times (10 + 2(3.5))$

$= 44 \times 0.5 \times (10 + 7) = 22 \times 17 = 374 \text{ cm}^2$

Hence, the total surface area of the article is 374 cm^2**.**

Exercise 12.2

Note: Unless stated otherwise, take $\pi = \frac{22}{7}$.

Q 1. **A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π.**

Radius of cone and hemisphere $(r) = 1 \text{ cm}$

Height of cone $(h) =$ Radius $(r) = 1 \text{ cm}$

Volume of the solid = (Volume of cone) + (Volume of hemisphere)

$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

$= \frac{1}{3}\pi(1)^2(1) + \frac{2}{3}\pi(1)^3$

$= \frac{1}{3}\pi + \frac{2}{3}\pi = \pi\left(\frac{1}{3} + \frac{2}{3}\right)$

$= \pi\left(\frac{3}{3}\right) = \pi \text{ cm}^3$

Hence, the volume of the solid is $\pi \text{ cm}^3$.

Q 2. **Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same.)**

Diameter of the model (cylinder and cones) $= 3 \text{ cm}$, so radius $(r) = 1.5 \text{ cm} = \frac{3}{2} \text{ cm}$

Total length of the model $= 12 \text{ cm}$

Height of each cone $(h_1) = 2 \text{ cm}$

Height of the cylindrical part $(h_2) =$ Total length $-2\times$ height of cone

$h_2 = 12 - 2(2) = 12 - 4 = 8 \text{ cm}$

Volume of air in the model = (Volume of cylinder) + 2 $\times$ (Volume of cone)

$= \pi r^2 h_2 + 2\left(\frac{1}{3}\pi r^2 h_1\right) = \pi r^2\left(h_2 + \frac{2}{3}h_1\right)$

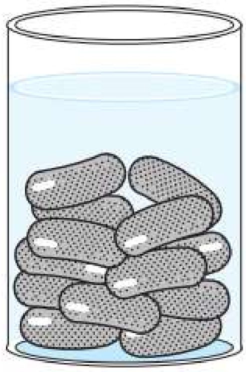
$= \frac{22}{7} \times \left(\frac{3}{2}\right)^2 \times \left(8 + \frac{2}{3}(2)\right)$

$= \frac{22}{7} \times \frac{9}{4} \times \left(8 + \frac{4}{3}\right) = \frac{11\times9}{14} \times \left(\frac{24+4}{3}\right)$

$= \frac{99}{14} \times \frac{28}{3} = 33 \times 2 = 66 \text{ cm}^3$

Hence, the volume of air contained in the model is 66 cm^3.

Q 3. **A gulab jamun, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm (see Fig. 12.15).**



आकृति 12.15

Diameter of a gulab jamun $= 2.8 \text{ cm}$, so radius $(r) = 1.4 \text{ cm}$

Total length of a gulab jamun $= 5 \text{ cm}$

Height of the cylindrical part $(h) =$ Total length $-2\times$ radius of hemisphere

$h = 5 - 2(1.4) = 5 - 2.8 = 2.2 \text{ cm}$

Volume of one gulab jamun = (Volume of cylinder) + 2 $\times$ (Volume of hemisphere)

$= \pi r^2 h + 2\left(\frac{2}{3}\pi r^3\right) = \pi r^2\left(h + \frac{4}{3}r\right)$

$= \frac{22}{7} \times (1.4)^2 \times \left(2.2 + \frac{4}{3}(1.4)\right)$

$= \frac{22}{7} \times 1.96 \times \left(2.2 + \frac{5.6}{3}\right) = 22 \times 0.28 \times \left(\frac{6.6+5.6}{3}\right)$

$= 6.16 \times \left(\frac{12.2}{3}\right) = \frac{75.152}{3} \text{ cm}^3$

Total volume of 45 gulab jamuns

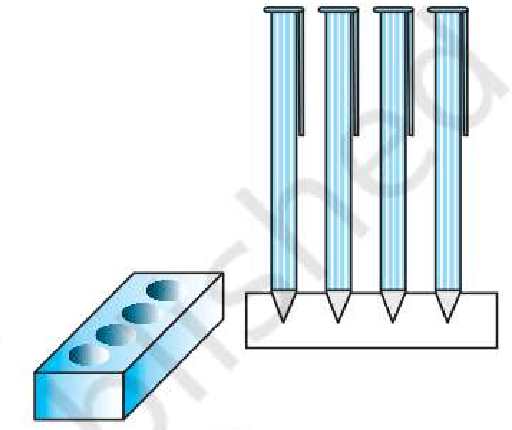
$= 45 \times \frac{75.152}{3} = 15 \times 75.152 = 1127.28 \text{ cm}^3$

Quantity of syrup = 30% of total volume $= \frac{30}{100} \times 1127.28$

$= 0.3 \times 1127.28 = 338.184 \text{ cm}^3 \approx 338 \text{ cm}^3$

Hence, the approximate quantity of syrup is 338 cm^3.

Q 4. **A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand (see Fig. 12.16).**



आकृति 12.16

Volume of the cuboidal wooden block

$= l \times b \times h = 15 \times 10 \times 3.5 = 525 \text{ cm}^3$

Radius of conical depression $(r) = 0.5 \text{ cm} = \frac{1}{2} \text{ cm}$

Depth of depression (height h_1) $= 1.4 \text{ cm}$

Volume of one conical depression $= \frac{1}{3}\pi r^2 h_1$

$= \frac{1}{3} \times \frac{22}{7} \times (0.5)^2 \times 1.4$

$= \frac{1}{3} \times \frac{22}{7} \times 0.25 \times 1.4 = \frac{1}{3} \times 22 \times 0.25 \times 0.2$

$= \frac{1}{3} \times 22 \times 0.05 = \frac{1.1}{3} \text{ cm}^3$

Total volume of 4 depressions $= 4 \times \frac{1.1}{3} = \frac{4.4}{3} = 1.47 \text{ cm}^3$ (approx)

Volume of remaining wood = (Volume of cuboid) - (Volume of 4 depressions)

$= 525 - 1.47 = 523.53 \text{ cm}^3$

Hence, the volume of wood in the entire stand is 523.53 cm^3**.**

Q 5. A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.

Radius of conical vessel $(R) = 5 \text{ cm}$ and height $(H) = 8 \text{ cm}$

Volume of vessel (cone) $= \frac{1}{3}\pi R^2 H$

$= \frac{1}{3}\pi(5)^2(8) = \frac{200}{3}\pi \text{ cm}^3$

Volume of water that flows out = $\frac{1}{4}$ of volume of vessel

$= \frac{1}{4} \times \frac{200}{3}\pi = \frac{50}{3}\pi \text{ cm}^3$

Radius of one lead shot (sphere) $(r) = 0.5 \text{ cm} = \frac{1}{2} \text{ cm}$

Volume of one lead shot $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{1}{2}\right)^3 = \frac{4}{3}\pi \times \frac{1}{8} = \frac{1}{6}\pi \text{ cm}^3$

Let the number of lead shots be n.

$n\times$ (Volume of one lead shot) = Volume of water that flows out

$n \times \frac{1}{6}\pi = \frac{50}{3}\pi$

$n = \frac{50}{3} \times \frac{6}{1} = 50 \times 2 = 100$

Hence, the number of lead shots dropped in the vessel is 100.

Q 6. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8g mass. (Use $\pi = 3.14$)

For the lower large cylinder:

Diameter $= 24$ cm, so radius $(R) = 12$ cm. Height $(H) = 220$ cm

Volume of large cylinder
$= \pi R^2 H = 3.14 \times (12)^2 \times 220 = 3.14 \times 144 \times 220 = 99475.2 \mathrm{~cm}^3$

For the upper small cylinder:

Radius $(r) = 8$ cm. Height $(h) = 60$ cm

Volume of small cylinder
$= \pi r^2 h = 3.14 \times (8)^2 \times 60 = 3.14 \times 64 \times 60 = 12057.6 \mathrm{~cm}^3$

Total volume of the pole $= 99475.2 + 12057.6 = 111532.8 \mathrm{~cm}^3$

Mass of 1 cm^3 iron = 8g

Total mass of the pole $= 111532.8 \times 8 = 892262.4$ g

Converting to kilograms $= \frac{892262.4}{1000} = 892.2624$ kg

Hence, the mass of the pole is 892.26 kg.

Q 7. A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.

Radius of cone $(r) = 60 \text{ cm}$, Height $(h_1) = 120 \text{ cm}$

Radius of hemisphere $(r) = 60 \text{ cm}$

Radius of cylinder $(R) = 60 \text{ cm}$, Height $(H) = 180 \text{ cm}$ (Since $r = R$, we will use just r)

Volume of the solid (cone + hemisphere) $= \frac{1}{3}\pi r^2 h_1 + \frac{2}{3}\pi r^3$

$= \frac{1}{3}\pi r^2(h_1 + 2r) = \frac{1}{3} \times \frac{22}{7} \times (60)^2 \times (120 + 2(60))$

$= \frac{1}{3} \times \frac{22}{7} \times 3600 \times 240 = \frac{22}{7} \times 1200 \times 240 = \frac{6336000}{7} \text{ cm}^3$

Volume of cylinder

$= \pi r^2 H = \frac{22}{7} \times (60)^2 \times 180 = \frac{22}{7} \times 3600 \times 180 = \frac{14256000}{7} \text{ cm}^3$

Volume of water left in the cylinder = (Volume of cylinder) - (Volume of the solid)

$= \frac{14256000}{7} - \frac{6336000}{7} = \frac{7920000}{7} \text{ cm}^3$

$= 1131428.57 \text{ cm}^3$

Converting to m^3 by dividing by 1000000 $(1 \text{ m} = 100 \text{ cm})$:

$= \frac{1131428.57}{1000000} \approx 1.131 \text{ m}^3$

Hence, the volume of water left is approximately 1.131 m^3**.**

Q 8. **A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm^3. Check whether she is correct, taking the above as the inside measurements, and $\pi = 3.14$.**

For cylindrical neck:

Diameter $= 2 \text{ cm}$, so radius $(r) = 1 \text{ cm}$

Length (height h) $= 8 \text{ cm}$

Volume of cylindrical part $= \pi r^2 h = 3.14 \times (1)^2 \times 8 = 25.12 \text{ cm}^3$

For spherical part:

Diameter $= 8.5 \text{ cm}$, so radius $(R) = \frac{8.5}{2} = 4.25 \text{ cm}$

Volume of spherical part $= \frac{4}{3}\pi R^3$

$= \frac{4}{3} \times 3.14 \times (4.25)^3$

$= \frac{4}{3} \times 3.14 \times 76.765625 = \frac{964.17625}{3} = 321.39 \text{ cm}^3$ (approx)

Total volume of vessel $= 25.12 + 321.39 = 346.51 \text{ cm}^3$

The child found the volume to be 345 cm^3, which is not correct.

Hence, the child's answer is incorrect. The correct volume is 346.51 cm^3.