

Chapter 11: Areas Related to Circles

Class 10 Math Chapter 11 Solutions (English Medium)

Exercise 11.1

Note: Unless stated otherwise, use $\pi = \frac{22}{7}$.

Q1. Find the area of a sector of a circle with radius 6 cm if angle of the sector is 60° .

Given: Radius of the circle (r) = 6 cm

Angle of the sector (θ) = 60°

$$\text{Area of the sector} = \frac{\theta}{360^\circ} \times \pi r^2$$

$$= \frac{60}{360} \times \frac{22}{7} \times (6)^2$$

$$= \frac{1}{6} \times \frac{22}{7} \times 36$$

$$= \frac{22}{7} \times 6 = \frac{132}{7} \text{ cm}^2$$

Hence, the area of the sector is $\frac{132}{7} \text{ cm}^2$.

Q 2. Find the area of a quadrant of a circle whose circumference is 22 cm.

Let the radius of the circle be r .

Circumference of the circle = $2\pi r = 22$ cm

$$2 \times \frac{22}{7} \times r = 22$$

$$r = \frac{22 \times 7}{2 \times 22} = \frac{7}{2} \text{ cm}$$

The angle of a quadrant of a circle is $\theta = \frac{360^\circ}{4} = 90^\circ$.

Area of quadrant = $\frac{1}{4} \times \pi r^2$

$$= \frac{1}{4} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$

$$= \frac{1}{4} \times \frac{11 \times 7}{2} = \frac{77}{8} \text{ cm}^2$$

Hence, the area of the quadrant is $\frac{77}{8} \text{ cm}^2$.

Q 3. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

Length of the minute hand acts as the radius of the circle, so $r = 14$ cm.

We know the minute hand completes one full circle (360°) in 60 minutes.

$$\text{Angle swept in 1 minute} = \frac{360^\circ}{60} = 6^\circ$$

$$\text{Angle swept in 5 minutes } (\theta) = 5 \times 6^\circ = 30^\circ$$

Area swept in 5 minutes = Area of sector with $\theta = 30^\circ$

$$= \frac{\theta}{360^\circ} \times \pi r^2$$

$$= \frac{30}{360} \times \frac{22}{7} \times 14 \times 14$$

$$= \frac{1}{12} \times 22 \times 2 \times 14$$

$$= \frac{1}{6} \times 22 \times 14 = \frac{1}{3} \times 11 \times 14 = \frac{154}{3} \text{ cm}^2$$

Hence, the area swept in 5 minutes is $\frac{154}{3} \text{ cm}^2$.

Q 4. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding :

(i) minor segment

(ii) major sector

(Use $\pi = 3.14$)

Given: Radius (r) = 10 cm, Angle subtended at centre (θ) = 90° .

(i) Area of minor segment:

Area of minor sector OAB =

$$\frac{\theta}{360^\circ} \times \pi r^2 = \frac{90}{360} \times 3.14 \times (10)^2 = \frac{1}{4} \times 3.14 \times 100 = \frac{314}{4} = 78.5 \text{ cm}^2$$

$$\text{Area of } \triangle OAB = \frac{1}{2} \times \text{Base} \times \text{Height} = \frac{1}{2} \times 10 \times 10 = 50 \text{ cm}^2 \text{ (Since } \angle O = 90^\circ)$$

Area of minor segment = Area of minor sector - Area of $\triangle OAB$

$$= 78.5 - 50 = 28.5 \text{ cm}^2$$

(ii) Area of major sector:

$$\text{Angle for major sector} = 360^\circ - 90^\circ = 270^\circ$$

Area of major sector =

$$\frac{270}{360} \times \pi r^2 = \frac{3}{4} \times 3.14 \times 100 = \frac{3}{4} \times 314 = 3 \times 78.5 = 235.5 \text{ cm}^2$$

Hence, the area of minor segment is 28.5 cm^2 and the area of major sector is 235.5 cm^2 .

- Q 5.** In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find:
- (i) the length of the arc
 - (ii) area of the sector formed by the arc
 - (iii) area of the segment formed by the corresponding chord

Given: Radius $r = 21$ cm, Angle $\theta = 60^\circ$

(i) Length of the arc:

$$\begin{aligned}\text{Length of arc} &= \frac{\theta}{360^\circ} \times 2\pi r = \frac{60}{360} \times 2 \times \frac{22}{7} \times 21 \\ &= \frac{1}{6} \times 2 \times 22 \times 3 = 22 \text{ cm}\end{aligned}$$

(ii) Area of the sector:

$$\begin{aligned}\text{Area of sector} &= \frac{\theta}{360^\circ} \times \pi r^2 = \frac{60}{360} \times \frac{22}{7} \times (21)^2 \\ &= \frac{1}{6} \times 22 \times 3 \times 21 = 11 \times 21 = 231 \text{ cm}^2\end{aligned}$$

(iii) Area of the segment:

Area of segment = Area of sector - Area of $\triangle OAB$

In $\triangle OAB$, $OA = OB = r$, so $\angle OAB = \angle OBA$. Since $\angle O = 60^\circ$, $\triangle OAB$ is an equilateral triangle.

$$\text{Area of equilateral triangle} = \frac{\sqrt{3}}{4} a^2 = \frac{\sqrt{3}}{4} \times (21)^2 = \frac{441\sqrt{3}}{4} \text{ cm}^2$$

$$\text{Hence, Area of segment} = \left(231 - \frac{441\sqrt{3}}{4} \right) \text{ cm}^2$$

Q 6. A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle.
(Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)

Radius $r = 15$ cm, Angle $\theta = 60^\circ$.

Area of minor segment:

$$\text{Area of sector} = \frac{60}{360} \times 3.14 \times (15)^2 = \frac{1}{6} \times 3.14 \times 225 = 117.75 \text{ cm}^2$$

$$\text{Area of equilateral } \triangle OAB = \frac{\sqrt{3}}{4} r^2 = \frac{1.73}{4} \times 225 = \frac{389.25}{4} = 97.3125 \text{ cm}^2$$

$$\text{Area of minor segment} = \text{Area of sector} - \text{Area of } \triangle OAB$$

$$= 117.75 - 97.3125 = 20.4375 \text{ cm}^2$$

Area of major segment:

$$\text{Area of full circle} = \pi r^2 = 3.14 \times 225 = 706.5 \text{ cm}^2$$

$$\text{Area of major segment} = \text{Area of circle} - \text{Area of minor segment}$$

$$= 706.5 - 20.4375 = 686.0625 \text{ cm}^2$$

Hence, the area of minor segment is 20.4375 cm^2 and area of major segment is 686.0625 cm^2 .

Q 7. A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle.
(Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)

Radius $r = 12$ cm, Angle $\theta = 120^\circ$.

Area of sector OAB =

$$\frac{120}{360} \times 3.14 \times (12)^2 = \frac{1}{3} \times 3.14 \times 144 = 3.14 \times 48 = 150.72 \text{ cm}^2$$

To find Area of $\triangle OAB$:

Draw perpendicular OM from centre O to chord AB. It bisects the angle 120° into two 60° angles and also bisects the chord.

In right $\triangle OMA$:

$$\sin 60^\circ = \frac{AM}{OA} \Rightarrow \frac{\sqrt{3}}{2} = \frac{AM}{12} \Rightarrow AM = 6\sqrt{3} \text{ cm}$$

So, Chord $AB = 2 \times AM = 12\sqrt{3}$ cm

$$\cos 60^\circ = \frac{OM}{OA} \Rightarrow \frac{1}{2} = \frac{OM}{12} \Rightarrow OM = 6 \text{ cm (Height)}$$

Area of $\triangle OAB =$

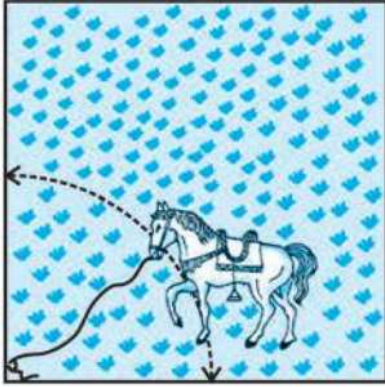
$$\frac{1}{2} \times \text{Base}(AB) \times \text{Height}(OM) = \frac{1}{2} \times 12\sqrt{3} \times 6 = 36\sqrt{3} = 36 \times 1.73 = 62.28 \text{ cm}^2$$

Area of corresponding segment = Area of sector - Area of $\triangle OAB$

$$= 150.72 - 62.28 = 88.44 \text{ cm}^2$$

Hence, the area of the corresponding segment is 88.44 cm^2 .

- Q 8.** A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope (see Fig. 11.8). Find:
- (i) the area of that part of the field in which the horse can graze.
 - (ii) the increase in the grazing area if the rope were 10 m long instead of 5 m. (Use $\pi = 3.14$)



आकृति 11.8

(i) Area grazed with a 5 m long rope:

Length of the rope acts as the radius of the sector, so $r = 5$ m.

The field is a square, so the angle at the corner is $\theta = 90^\circ$.

$$\begin{aligned}\text{Area the horse can graze} &= \text{Area of quadrant} = \frac{\theta}{360^\circ} \times \pi r^2 \\ &= \frac{90}{360} \times 3.14 \times (5)^2 = \frac{1}{4} \times 3.14 \times 25 = \frac{78.5}{4} = 19.625 \text{ m}^2\end{aligned}$$

(ii) Increase in grazing area if rope is 10 m:

New radius $R = 10$ m. Angle $\theta = 90^\circ$.

$$\text{New grazing area} = \frac{90}{360} \times 3.14 \times (10)^2 = \frac{1}{4} \times 3.14 \times 100 = \frac{314}{4} = 78.5 \text{ m}^2$$

Increase in area = New area - Old area

$$= 78.5 - 19.625 = 58.875 \text{ m}^2$$

Hence, the increase in grazing area is 58.875 m².

Q 9. A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in Fig. 11.9. Find:

- (i) the total length of the silver wire required.
- (ii) the area of each sector of the brooch.



आकृति 11.9

Given: Diameter $d = 35$ mm, so radius $r = \frac{35}{2}$ mm.

Number of diameters = 5.

(i) Total length of silver wire required:

Length of wire = Circumference of circle + Length of 5 diameters

$$= \pi d + 5d = \left(\frac{22}{7} \times 35\right) + (5 \times 35)$$

$$= (22 \times 5) + 175 = 110 + 175 = 285 \text{ mm}$$

(ii) Area of each sector:

Since the circle is divided into 10 equal sectors, angle of each sector $\theta = \frac{360^\circ}{10} = 36^\circ$.

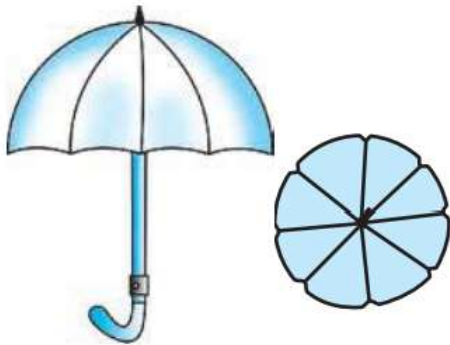
$$\text{Area of each sector} = \frac{1}{10} \times \pi r^2$$

$$= \frac{1}{10} \times \frac{22}{7} \times \left(\frac{35}{2}\right) \times \left(\frac{35}{2}\right)$$

$$= \frac{1}{10} \times \frac{22}{7} \times \frac{1225}{4} = \frac{1}{10} \times \frac{22 \times 175}{4} = \frac{1}{10} \times \frac{3850}{4} = \frac{385}{4} \text{ mm}^2$$

Hence, the total length of wire is 285 mm and area of each sector is $\frac{385}{4}$ mm².

- Q 10.** An umbrella has 8 ribs which are equally spaced (see Fig. 11.10). Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.



आकृति 11.10

The umbrella is assumed to be a flat circle of radius $r = 45$ cm.

It is divided into 8 equal parts by the ribs.

Angle between two consecutive ribs $\theta = \frac{360^\circ}{8} = 45^\circ$.

Area between two consecutive ribs (Area of sector) = $\frac{\theta}{360^\circ} \times \pi r^2$

$$= \frac{45}{360} \times \frac{22}{7} \times (45)^2 = \frac{1}{8} \times \frac{22}{7} \times 2025$$

$$= \frac{1}{4} \times \frac{11}{7} \times 2025 = \frac{22275}{28} \text{ cm}^2$$

Hence, the area between two consecutive ribs is $\frac{22275}{28}$ cm².

- Q 11.** A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115° . Find the total area cleaned at each sweep of the blades.

Length of each wiper blade acts as radius $r = 25$ cm.

Angle of sweep $\theta = 115^\circ$.

Area cleaned by one wiper = Area of sector = $\frac{115}{360} \times \pi r^2$

Since there are 2 wipers, total area cleaned = $2 \times \frac{115}{360} \times \frac{22}{7} \times (25)^2$

$$= 2 \times \frac{23}{72} \times \frac{22}{7} \times 625 = \frac{23}{36} \times \frac{22}{7} \times 625 = \frac{23}{18} \times \frac{11}{7} \times 625$$

$$= \frac{23 \times 11 \times 625}{18 \times 7} = \frac{158125}{126} \text{ cm}^2$$

Hence, the total area cleaned at each sweep is $\frac{158125}{126}$ cm².

Q 12. To warn ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned. (Use $\pi = 3.14$)

Distance over which light spreads is the radius $r = 16.5$ km.

Angle of sector $\theta = 80^\circ$.

$$\text{Area over which light spreads} = \frac{\theta}{360^\circ} \times \pi r^2$$

$$= \frac{80}{360} \times 3.14 \times (16.5)^2$$

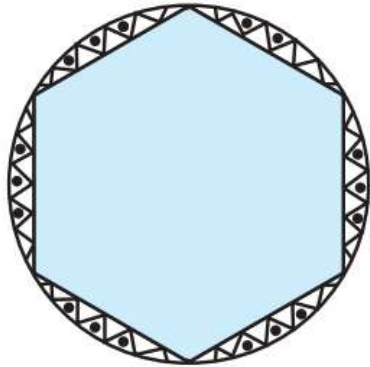
$$= \frac{2}{9} \times 3.14 \times 272.25$$

$$= 2 \times 3.14 \times 30.25$$

$$= 6.28 \times 30.25 = 189.97 \text{ km}^2$$

Hence, the area of the sea over which ships are warned is 189.97 km^2 .

- Q 13.** A round table cover has six equal designs as shown in Fig. 11.11. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of Rs 0.35 per cm^2 . (Use $\sqrt{3} = 1.7$)



आकृति 11.11

Radius of the table cover $r = 28$ cm.

Since there are 6 equal designs, angle subtended by each design at the centre $\theta = \frac{360^\circ}{6} = 60^\circ$.

Area of one design = Area of a segment

= Area of sector - Area of equilateral \triangle

$$= \frac{60}{360} \times \frac{22}{7} \times (28)^2 - \frac{\sqrt{3}}{4} \times (28)^2$$

$$= \frac{1}{6} \times \frac{22}{7} \times 784 - \frac{1.7}{4} \times 784$$

$$= \frac{1}{6} \times 22 \times 112 - 1.7 \times 196$$

$$= \frac{2464}{6} - 333.2 = 410.66 - 333.2 = 77.46 \text{ cm}^2 \text{ (approx)}$$

$$\text{Total area of 6 designs} = 6 \times \left(\frac{2464}{6} - 333.2 \right) = 2464 - 1999.2 = 464.8 \text{ cm}^2$$

$$\text{Cost} = 0.35 \times 464.8 = \text{Rs } 162.68$$

Hence, the cost of making the designs is Rs 162.68.

Q 14. Tick the correct answer in the following:

Area of a sector of angle p° (in degrees) of a circle with radius R is

(A) $\frac{p}{180} \times 2\pi R$

(B) $\frac{p}{180} \times \pi R^2$

(C) $\frac{p}{360} \times 2\pi R$

(D) $\frac{p}{720} \times 2\pi R^2$

We know that the area of a sector of angle θ and radius r is given by $\frac{\theta}{360^\circ} \times \pi r^2$.

Here $\theta = p^\circ$ and $r = R$.

So, Area = $\frac{p}{360} \times \pi R^2$

Looking at option (D): $\frac{p}{720} \times 2\pi R^2 = \frac{p \times 2}{360 \times 2} \times \pi R^2 = \frac{p}{360} \times \pi R^2$

Hence, the correct option is (D) $\frac{p}{720} \times 2\pi R^2$.