Chapter 10: Circles

Class 10 Math Chapter 10 Solutions (English Medium)

Exercise 10.1

Q 1. **How many tangents can a circle have?**

A circle can have **infinitely many** tangents.

Reason: A circle is made up of infinite points on its circumference, and at each point, one tangent can be drawn.

Q 2. **Fill in the blanks:**
(i) A tangent to a circle intersects it in ______ point (s).
(ii) A line intersecting a circle in two points is called a ______.
(iii) A circle can have ______ parallel tangents at the most.
(iv) The common point of a tangent to a circle and the circle is called ______.

(i) A tangent to a circle intersects it in **one** point.

(ii) A line intersecting a circle in two points is called a **secant**.

(iii) A circle can have **two** parallel tangents at the most.

(iv) The common point of a tangent to a circle and the circle is called the **point of contact**.

Q 3. **A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Length PQ is :**
(A) 12 cm
(B) 13 cm
(C) 8.5 cm
(D) $\sqrt{119}$ cm

Given: Radius of the circle OP = 5 cm.

Distance from the centre O to point Q, OQ = 12 cm.

We know that the radius through the point of contact is perpendicular to the tangent.

Therefore, $\angle OPQ = 90^\circ$.

In right $\triangle OPQ$, by Pythagoras theorem:

$OQ^2 = OP^2 + PQ^2$

$(12)^2 = (5)^2 + PQ^2$

$144 = 25 + PQ^2$

$PQ^2 = 144 - 25 = 119$

$PQ = \sqrt{119}\ \text{cm}$

Hence, the correct option is (D) $\sqrt{119}\ \text{cm}$.

Q 4. **Draw a circle and two lines parallel to a given line such that one is a tangent and the other, a secant to the circle.**

Construction Steps:

1. Draw a circle with centre O and any suitable radius.

2. Draw a given line l outside the circle.

3. Draw a perpendicular from centre O to the line l intersecting the circle at point P and line l at M.

4. At point P, draw a line m parallel to line l. Since line m touches the circle at exactly one point P, it is the **tangent**.

5. Draw another line n parallel to line l passing through the interior of the circle. This line intersects the circle at two points, so it is the **secant**.

Exercise 10.2

Q 1. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is:
(A) 7 cm
(B) 12 cm
(C) 15 cm
(D) 24.5 cm

Let O be the centre of the circle and P be the point of contact.

Length of tangent PQ = 24 cm.

Distance from centre O to Q, OQ = 25 cm.

Radius OP is perpendicular to tangent PQ (Theorem 10.1). So, $\angle OPQ = 90^\circ$.

In right $\triangle OPQ$, by Pythagoras theorem:

$OQ^2 = OP^2 + PQ^2$

$(25)^2 = OP^2 + (24)^2$

$625 = OP^2 + 576$

$OP^2 = 625 - 576 = 49$

$OP = \sqrt{49} = 7 \text{ cm}$

Hence, the correct option is (A) 7 cm.

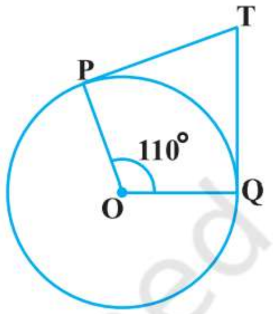
Q 2. **In Fig. 10.11, if TP and TQ are the two tangents to a circle with centre O so that $\angle POQ = 110^\circ$, then $\angle PTQ$ is equal to:**

(A) 60°

(B) 70°

(C) 80°

(D) 90°



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We know that the radius is perpendicular to the tangent at the point of contact.

Therefore, $\angle OPT = 90^\circ$ and $\angle OQT = 90^\circ$.

In quadrilateral POQT, the sum of all angles is 360°:

$\angle POQ + \angle OPT + \angle OQT + \angle PTQ = 360^\circ$

$110^\circ + 90^\circ + 90^\circ + \angle PTQ = 360^\circ$

$290^\circ + \angle PTQ = 360^\circ$

$\angle PTQ = 360^\circ - 290^\circ = 70^\circ$

Hence, the correct option is (B) 70°**.**

Q 3. **If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80°, then $\angle POA$ is equal to:**

(A) 50°

(B) 60°

(C) 70°

(D) 80°

Tangents PA and PB are inclined to each other at 80°, so $\angle APB = 80^\circ$.

Radius is perpendicular to the tangent, so $\angle OAP = 90^\circ$ and $\angle OBP = 90^\circ$.

In quadrilateral OAPB: $\angle AOB + \angle OAP + \angle OBP + \angle APB = 360^\circ$

$\angle AOB + 90^\circ + 90^\circ + 80^\circ = 360^\circ$

$\angle AOB + 260^\circ = 360^\circ \Rightarrow \angle AOB = 100^\circ$

Now, in $\triangle OAP$ and $\triangle OBP$, the line joining the centre to the external point bisects the angle between the tangents and also the angle at the centre.

Therefore, $\angle POA = \frac{1}{2}\angle AOB = \frac{100^\circ}{2} = 50^\circ$.

Hence, the correct option is (A) 50°**.**

Q 4. **Prove that the tangents drawn at the ends of a diameter of a circle are parallel.**

Proof:

Let there be a circle with centre O and AB be its diameter.

Let PQ be the tangent drawn at point A and RS be the tangent drawn at point B.

We know that the tangent at any point of a circle is perpendicular to the radius through the point of contact (Theorem 10.1).

Therefore, $OA \perp PQ$ and $OB \perp RS$.

Thus, $\angle OAP = 90^\circ$ and $\angle OBS = 90^\circ$.

Since AB is a straight line (diameter), $\angle OAP$ and $\angle OBS$ form alternate interior angles for the lines PQ and RS.

Here, $\angle OAP = \angle OBS = 90^\circ$.

Since alternate interior angles are equal, the lines PQ and RS are parallel $(PQ \parallel RS)$.

Hence Proved.

Q 5. **Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.**

Proof by Contradiction:

Let there be a circle with centre O and AB is a tangent at point P on the circle.

We need to prove that the perpendicular on AB at P passes through the centre O.

Let us assume that the perpendicular at P to AB does not pass through O, but passes through another point O'.

Therefore, $\angle O'PB = 90^\circ$ ... (1) (By our assumption)

But we know from Theorem 10.1 that the radius joining the centre to the point of contact is perpendicular to the tangent.

Therefore, $OP \perp AB$, which means $\angle OPB = 90^\circ$ ... (2)

From equations (1) and (2), we get: $\angle O'PB = \angle OPB = 90^\circ$

This is only possible if the line O'P and OP coincide (i.e., they are the same line).

Hence, our assumption that the perpendicular passes through another point O' is wrong.

Conclusion: The perpendicular at the point of contact to the tangent passes through the centre.

Q 6. **The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.**

Let O be the centre of the circle and B be the point of contact.

Distance from centre O to A, OA = 5 cm.

Length of tangent AB = 4 cm.

We know that the radius is perpendicular to the tangent $(OB \perp AB)$. So, $\angle OBA = 90°$.

In right $\triangle OBA$, by Pythagoras theorem:

$OA^2 = OB^2 + AB^2$

$(5)^2 = OB^2 + (4)^2$

$25 = OB^2 + 16 \Rightarrow OB^2 = 25 - 16 = 9$

$OB = \sqrt{9} = 3 \text{ cm}$

Hence, the radius of the circle is 3 cm.

Q 7. **Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.**

Let O be the centre of the two concentric circles.

Radius of larger circle OA = 5 cm and radius of smaller circle OP = 3 cm.

Let AB be the chord of the larger circle which touches the smaller circle at point P.

Since AB is a tangent to the smaller circle and OP is the radius, $OP \perp AB$ (Theorem 10.1).

In right $\triangle OPA$, by Pythagoras theorem:

$OA^2 = OP^2 + AP^2$

$(5)^2 = (3)^2 + AP^2$

$25 = 9 + AP^2 \Rightarrow AP^2 = 25 - 9 = 16$

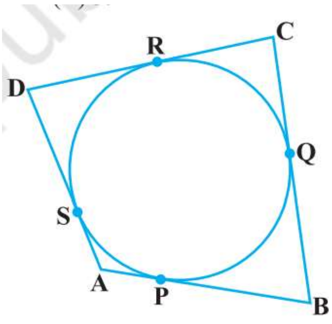
$AP = \sqrt{16} = 4 \text{ cm}$

We know that the perpendicular drawn from the centre to a chord bisects the chord.

Therefore, total length of chord AB = $2 \times AP = 2 \times 4 = 8 \text{ cm.}$

Hence, the length of the chord is 8 cm.

Q 8. **A quadrilateral ABCD is drawn to circumscribe a circle (see Fig. 10.12). Prove that** $AB + CD = AD + BC$



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Proof:

Since the lengths of tangents drawn from an external point to a circle are equal (Theorem 10.2), we have:

Tangents from point A: $AP = AS$ ... (1)

Tangents from point B: $BP = BQ$ ... (2)

Tangents from point C: $CR = CQ$ ... (3)

Tangents from point D: $DR = DS$ ... (4)

Adding equations (1), (2), (3) and (4):

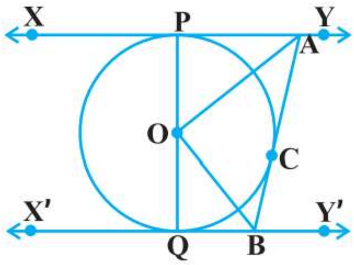
$(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)$

Since $AP + BP = AB$, $CR + DR = CD$, $AS + DS = AD$, and $BQ + CQ = BC$,

Therefore, $AB + CD = AD + BC$

Hence Proved.

Q 9. **In Fig. 10.13, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X'Y' at B. Prove that $\angle AOB = 90°$.**



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Proof:

Construction: Join OC.

Now, in $\triangle OPA$ and $\triangle OCA$:

$OP = OC$ (Radii of the same circle)

$OA = OA$ (Common)

$AP = AC$ (Tangents from external point A)

Thus, by SSS congruence criterion: $\triangle OPA \cong \triangle OCA$

Therefore, $\angle POA = \angle COA$ ... (1)

Similarly, in $\triangle OQB$ and $\triangle OCB$:

$\triangle OQB \cong \triangle OCB$

Therefore, $\angle QOB = \angle COB$ ... (2)

Since POQ is a diameter and a straight line:

$\angle POA + \angle COA + \angle COB + \angle QOB = 180°$

Substituting values from (1) and (2):

$2\angle COA + 2\angle COB = 180°$

$2(\angle COA + \angle COB) = 180°$

$\angle COA + \angle COB = 90°$

Since $\angle COA + \angle COB = \angle AOB$,

Thus, $\angle AOB = 90°$

Hence Proved.

Q 10. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

Proof:

Let there be a circle with centre O and an external point P from which two tangents PA and PB are drawn.

A and B are the points of contact. We need to prove that $\angle APB + \angle AOB = 180^\circ$.

We know that the radius is perpendicular to the tangent, so $\angle OAP = 90^\circ$ and $\angle OBP = 90^\circ$.

In quadrilateral OAPB, the sum of all angles is 360°:

$\angle OAP + \angle OBP + \angle APB + \angle AOB = 360^\circ$

$90^\circ + 90^\circ + \angle APB + \angle AOB = 360^\circ$

$180^\circ + \angle APB + \angle AOB = 360^\circ$

$\angle APB + \angle AOB = 360^\circ - 180^\circ = 180^\circ$

Since the sum of these two angles is 180°, they are supplementary.

Hence Proved.

Q 11. Prove that the parallelogram circumscribing a circle is a rhombus.

Proof:

Let ABCD be a parallelogram circumscribing a circle and touching it at points P, Q, R, S.

Since ABCD is a parallelogram, opposite sides are equal:

$AB = CD$ and $AD = BC$ ... (1)

We know that tangents drawn from an external point are equal:

$AP = AS$

$BP = BQ$

$CR = CQ$

$DR = DS$

Adding these four equations (similar to Question 8):

$(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)$

$AB + CD = AD + BC$ ... (2)

Substituting CD with AB and BC with AD from equation (1):

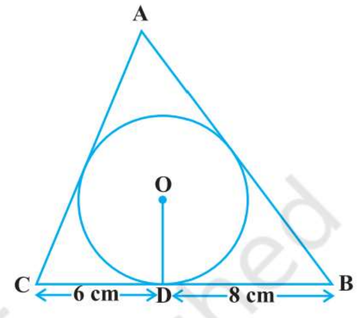
$AB + AB = AD + AD$

$2AB = 2AD \Rightarrow AB = AD$

Now since $AB = CD$ and $AB = AD$, all four sides are equal: $AB = BC = CD = DA$.

Hence, ABCD is a rhombus. Proved.

Q 12. **A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see Fig. 10.14). Find the sides AB and AC.**



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Let the circle touch $\triangle ABC$ at points D, E and F.

Radius of the circle $OD = OE = OF = 4$ cm.

Tangents from an external point are equal:

$CD = CE = 6$ cm

$BD = BF = 8$ cm

Let $AF = AE = x$ cm.

Sides are: $a = BC = 6 + 8 = 14$ cm, $b = AC = 6 + x$ cm, $c = AB = 8 + x$ cm.

Semi-perimeter $s = \frac{a+b+c}{2} = \frac{14+(6+x)+(8+x)}{2} = \frac{28+2x}{2} = 14 + x$

Area of $\triangle ABC$ by Heron's Formula:

$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$

$\text{Area} = \sqrt{(14+x)(14+x-14)(14+x-6-x)(14+x-8-x)}$

$\text{Area} = \sqrt{(14+x)(x)(8)(6)} = \sqrt{48x(14+x)}$ ... (1)

Alternatively, Area of $\triangle ABC$ is the sum of areas of 3 smaller triangles:

$\text{Area}(\triangle ABC) = \text{Area}(\triangle OBC) + \text{Area}(\triangle OCA) + \text{Area}(\triangle OAB)$

$\text{Area} = \frac{1}{2} \times BC \times OD + \frac{1}{2} \times AC \times OE + \frac{1}{2} \times AB \times OF$

Since $OD = OE = OF = 4$:

$\text{Area} = \frac{1}{2} \times 4 \times (BC + AC + AB) = 2 \times (14 + 6 + x + 8 + x) = 2(28 + 2x) = 56 + 4x = 4(14 + x$ ... (2)

Equating (1) and (2):

$\sqrt{48x(14+x)} = 4(14+x)$

Squaring both sides:

$48x(14+x) = 16(14+x)^2$

$3x = 14 + x$ (Since $14 + x \neq 0$)

$2x = 14 \Rightarrow x = 7$ cm

Therefore, the sides are:

$AB = 8 + x = 8 + 7 = 15$ cm

$AC = 6 + x = 6 + 7 = 13 \text{ cm}$

Q 13. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Proof:

Let ABCD be a quadrilateral circumscribing a circle with centre O, touching the circle at P, Q, R, S.

Join centre O to P, Q, R, S and to A, B, C, D.

This forms 8 angles at the centre: $\angle 1, \angle 2, \angle 3, \angle 4, \angle 5, \angle 6, \angle 7, \angle 8.$

We know that tangents drawn from an external point subtend equal angles at the centre.

Therefore: $\angle 1 = \angle 2$, $\angle 3 = \angle 4$, $\angle 5 = \angle 6$, and $\angle 7 = \angle 8$

Since the sum of all angles at the centre is 360°:

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ$

Substituting equal angles:

$2\angle 1 + 2\angle 4 + 2\angle 5 + 2\angle 8 = 360^\circ \Rightarrow \angle 1 + \angle 4 + \angle 5 + \angle 8 = 180^\circ$

And $2\angle 2 + 2\angle 3 + 2\angle 6 + 2\angle 7 = 360^\circ \Rightarrow \angle 2 + \angle 3 + \angle 6 + \angle 7 = 180^\circ$

Here $\angle AOB = \angle 1 + \angle 8$ and $\angle COD = \angle 4 + \angle 5$,

So $\angle AOB + \angle COD = (\angle 1 + \angle 8) + (\angle 4 + \angle 5) = 180^\circ$.

Similarly $\angle BOC + \angle AOD = 180^\circ$.

Since the sum of angles subtended by opposite sides is 180°, they are supplementary.

Hence Proved.