Chapter 1: Real Numbers - Important Questions and Answers

Exercise 1.1

Question 1: Express each number as a product of its prime factors:

- 140: $2^2 \times 5 \times 7$
- 156: $2^2 \times 3 \times 13$
- 3825: $3^2 \times 5^2 \times 17$
- 5005: $5 \times 7 \times 11 \times 13$
- 7429: $17 \times 19 \times 23$

Question 2: Find the LCM and HCF of the following pairs of integers and verify that $\text{Product of two numbers} = \text{HCF} \times \text{LCM}$.

- 26 and 91:
  - $26 = 2 \times 13$
  - $91 = 7 \times 13$
  - $HCF = 13; LCM = 182$
  - Verification: $26 \times 91 = 2366$; $13 \times 182 = 2366$. (True)
- 510 and 92:
  - $510 = 2 \times 3 \times 5 \times 17$
  - $92 = 2^2 \times 23$
  - HCF = 2; LCM = 23460
  - Verification: $510 \times 92 = 46920$; $2 \times 23460 = 46920$. (True)
- 336 and 54:
  - $336 = 2^4 \times 3 \times 7$
  - $54 = 2 \times 3^3$
  - HCF = 6; LCM = 3024
  - Verification: $336 \times 54 = 18144$; $6 \times 3024 = 18144$. (True)

Question 3: Find the HCF and LCM of the following integers by applying the prime factorization method:

- 12, 15, and 21: HCF = 3; LCM = 420
- 17, 23, and 29: HCF = 1; LCM = 11339
- 8, 9, and 25: HCF = 1; LCM = 1800

Question 4: Given that HCF(306, 657) = 9, find LCM(306, 657).

- Formula: $\text{LCM} \times \text{HCF} = \text{Product of two numbers}$
- Calculation: $\text{LCM} = \frac{306 \times 657}{9} = 22338$

Question 5: Check whether 6^n can end with the digit 0 for any natural number n.

- Answer: No.
- Reason: For a number to end with the digit 0, its prime factorization must contain both 2 and 5. The prime factorization of 6^n is $(2 \times 3)^n$. Since it does not contain the factor 5, it can never end with the digit 0.

Exercise 1.2: Irrational Numbers

Question 1: Prove that $\sqrt{5}$ is an irrational number.

- Solution: Let us assume, to the contrary, that $\sqrt{5}$ is a rational number.
    - Therefore, we can write $\sqrt{5} = \frac{a}{b}$, where a and b are co-prime integers and $b \neq 0$.
    - Squaring both sides: $5 = \frac{a^2}{b^2} \Rightarrow a^2 = 5b^2$.
    - This means a^2 is divisible by 5, which implies that a is also divisible by 5.
    - Now, let $a = 5c$ for some integer c.
    - Substituting this value: $(5c)^2 = 5b^2 \Rightarrow 25c^2 = 5b^2 \Rightarrow b^2 = 5c^2$.
    - This means b^2 is divisible by 5, so b is also divisible by 5.
    - Since a and b both have 5 as a common factor, it contradicts our assumption that a and b are co-prime.
    - Conclusion: Our assumption was wrong. Therefore, $\sqrt{5}$ is irrational.

Question 2: Prove that $3 + 2\sqrt{5}$ is an irrational number.

- Solution: Let us assume that $3 + 2\sqrt{5}$ is a rational number.
    - We can write: $3 + 2\sqrt{5} = \frac{a}{b}$ (where a, b are integers and $b \neq 0$).
    - Rearranging the equation: $2\sqrt{5} = \frac{a}{b} - 3 \Rightarrow 2\sqrt{5} = \frac{a-3b}{b}$
    - $\sqrt{5} = \frac{a-3b}{2b}$
    - Since a and b are integers, $\frac{a-3b}{2b}$ is a rational number.

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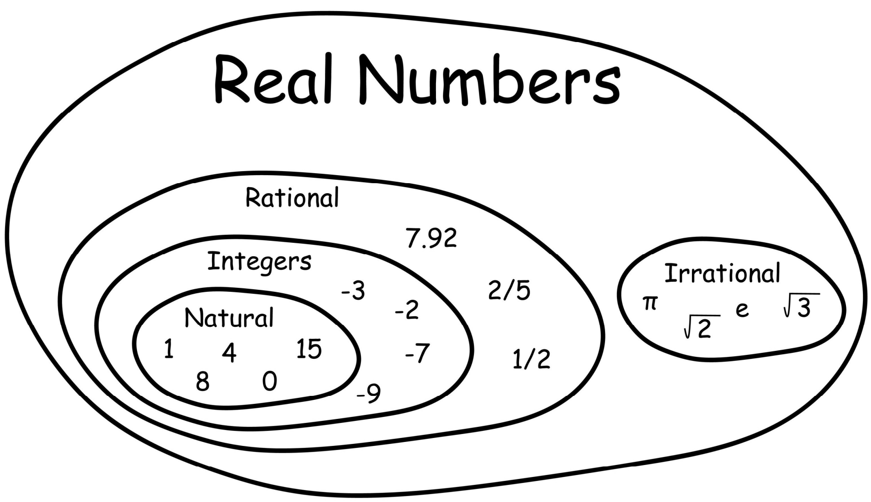
- However, we know that $\sqrt{5}$ is irrational.
- This contradiction has arisen because of our incorrect assumption.
- Conclusion: $3 + 2\sqrt{5}$ is irrational.

Question 3: Prove that the following numbers are irrational:

- $\frac{1}{\sqrt{2}}$: * Assume $\frac{1}{\sqrt{2}}$ is rational, so $\frac{1}{\sqrt{2}} = \frac{a}{b}$.
  - Inverting gives $\sqrt{2} = \frac{b}{a}$. Since b/a is rational but $\sqrt{2}$ is irrational, this is a contradiction. Thus, $\frac{1}{\sqrt{2}}$ is irrational.
- $7\sqrt{5}$: * Assume $7\sqrt{5} = \frac{a}{b}$. Then $\sqrt{5} = \frac{a}{7b}$.
  - Since $a/7b$ is rational but $\sqrt{5}$ is irrational, this is a contradiction. Thus, $7\sqrt{5}$ is irrational.
- $6 + \sqrt{2}$: * Assume $6 + \sqrt{2} = \frac{a}{b}$. Then $\sqrt{2} = \frac{a}{b} - 6$.
  - Since the RHS is rational but $\sqrt{2}$ is irrational, this is a contradiction. Thus, $6 + \sqrt{2}$ is irrational.



Previous Board Exam Questions and Important Potential Questions

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Part 1: Very Short Answer Questions (1 Mark)

Question 1: Express the number 156 as a product of its prime factors. [CBSE 2020, 2023]

- Answer: $156 = 2^2 \times 3 \times 13$

Question 2: If the HCF of two numbers is 9 and their product is 3060, find the LCM. [CBSE2019]

- Answer: * Formula: $\text{LCM} \times \text{HCF} = \text{Product of two numbers}$
  - $\text{LCM} = \frac{3060}{9} = 340$

Question 3: Can the number 4^n ever end with the digit 0? Give a reason. [CBSE 2018, 2022]

- Answer: No.
  - For a number to end with the digit 0, its prime factorization must contain both 2 and 5. The prime factorization of 4^n is $(2^2)^n$, which contains only the prime factor 2 and not 5.

Question 4: What is the difference between a prime number and a composite number? [New Potential Question]

- Answer: A prime number has exactly two factors (1 and itself), whereas a composite number has more than two factors.

Part 2: Short Answer Questions (2-3 Marks)

Question 5: Find the HCF and LCM of 96 and 404 using the prime factorization method. [CBSE 2020, RBSE 2022]

- Answer:
  - $96 = 2^5 \times 3$
  - $404 = 2^2 \times 101$
  - HCF: $2^2 = 4$
  - LCM: $2^5 \times 3 \times 101 = 9696$

Question 6: Explain why $(7 \times 11 \times 13 + 13)$ and $(7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5)$ are composite numbers? [CBSE 2023, NCERT]

- Answer: * First number: $13(7 \times 11 + 1) = 13 \times 78$.
  - Second number: $5(7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1) = 5 \times 1009$.
  - Since both numbers can be expressed as a product of factors other than 1 and themselves (i.e., they have more than two factors), they are composite numbers.

Question 7: Prove that $\sqrt{3}$ is an irrational number. [CBSE 2016, 2021, RBSE 2024]

- Tip: This is proved using the 'Method of Contradiction'. This is one of the most important questions for the exam.

Question 8: Find the largest number which divides 70 and 125, leaving remainders 5 and 8 respectively. [Potential Question]

- Answer:
  - The required numbers are: $70 - 5 = 65$ and $125 - 8 = 117$.
  - Now, find the HCF of 65 and 117:

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- $65 = 5 \times 13$
- $117 = 3^2 \times 13$
- HCF = 13. Therefore, the largest number is 13.

Part 3: Long Answer Questions (4-5 Marks)

Question 9: Prove that $5 - \sqrt{3}$ is an irrational number. [CBSE 2018, 2022, RBSE 2023]

- Answer: Let $5 - \sqrt{3}$ be a rational number $\frac{a}{b}$ (where a, b are integers and $b \neq 0$).
  - $\sqrt{3} = 5 - \frac{a}{b} = \frac{5b-a}{b}$
  - Since a and b are integers, $\frac{5b-a}{b}$ is rational. However, we know that $\sqrt{3}$ is irrational.
  - A rational number can never equal an irrational number. Therefore, our assumption was wrong, and $5 - \sqrt{3}$ is irrational.

Question 10: Circular Path Question (Sonia and Ravi): [CBSE 2017, 2019]

- Problem: Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. If they start at the same point and time, after how many minutes will they meet again at the starting point?
- Answer: We need to find the LCM of 18 and 12.
  - $18 = 2 \times 3^2$
  - $12 = 2^2 \times 3$
  - LCM $= 2^2 \times 3^2 = 4 \times 9 = 36$.
  - They will meet again at the starting point after 36 minutes.

Question 11: Show that any positive odd integer is of the form $6q + 1$, $6q + 3$, or $6q + 5$. [NCERT]

- Answer: Using Euclid's Division Lemma ($a = bq + r$), where $b = 6$ and $0 \leq r < 6$. For an integer to be odd, the remainder r must be 1, 3, or 5.

Important Potential Questions for the Future

Part 1: Objective Questions (MCQs / 1 Mark)

Question 1: If two positive integers p and q are expressed as $p = ab^2$ and $q = a^3b$ (where a, b are prime numbers), then what is the LCM(p, q)?

- Answer: a^3b^2 (In LCM, we take the highest power of each prime factor).

Question 2: Find the smallest number that is divisible by all numbers from 1 to 10 (both inclusive).

- Answer: 2520 (This is the LCM of all numbers from 1 to 10).

Question 3: What is the value of HCF$(a, b) \times$ LCM(a, b)?

- Answer: $a \times b$ (The product of the two numbers).

Part 2: Short Answer Questions (2-3 Marks)

Question 4: Prove that $\sqrt{p}$ is an irrational number, where p is a prime number.

- Importance: This question can be adapted for $\sqrt{2}, \sqrt{3}, or \sqrt{7}$. Practice solving this using the 'Method of Contradiction'.

Question 5: Without actually performing the long division, state whether the decimal expansion of $\frac{13}{3125}$ is terminating or non-terminating repeating.

- Answer: Factorizing the denominator: $3125 = 5^5$. Since the denominator is of the form $2^n \times 5^m$ (where $n = 0, m = 5$), its decimal expansion is terminating.

Question 6: Find the HCF and LCM of 12, 15, and 21 using the prime factorization method.

- Answer:
  - $12 = 2^2 \times; 15 = 3 \times; 21 = 3$
  - HCF = 3; LCM = 420

Question 7: Check whether 6^n can end with the digit 0 or 5 for any natural number n.

- Answer: No. $6^n = (2 \times 3)^n$. Its factors do not include 5, so it can never end in 0 or 5.

Part 3: Long Answer Questions (4-5 Marks)

Question 8: Prove that $3 + 5\sqrt{2}$ is an irrational number, given that $\sqrt{2}$ is irrational.

- Step 1 (Assumption): Assume $3 + 5\sqrt{2}$ is a rational number $\frac{a}{b}$ (where a, b are co-prime and $b \neq 0$).
- Step 2 (Rearranging): $5\sqrt{2} = \frac{a}{b} - 3 \Rightarrow 5\sqrt{2} = \frac{a-3b}{b} \Rightarrow \sqrt{2} = \frac{a-3b}{5b}$
- Step 3 (Logic): Since a, b, 3, and 5 are integers, $\frac{a-3b}{5b}$ is rational. This implies $\sqrt{2}$ is rational.
- Step 4 (Contradiction): This contradicts the fact that $\sqrt{2}$ is irrational. Therefore, $3 + 5\sqrt{2}$ is irrational.

Question 9: Three runners complete a lap in 20, 30, and 45 minutes respectively. If they start together, after how much time will they meet again at the starting point?

- Answer: Find the LCM of 20, 30, and 45.
  - $20 = 2^2 \times 5; 30 = 2 \times 3 \times 5; 45 = 3^2 \times 5$
  - LCM = $2^2 \times 3^2 \times 5 = 180$ minutes (or 3 hours).

Question 10: State the Fundamental Theorem of Arithmetic and use it to find the HCF of 504 and 980.

- Answer: Statement: "Every composite number can be expressed as a product of primes, and this factorization is unique."
  - $504 = 2^3 \times 3^2 \times 7; 980 = 2^2 \times 5 \times 7^2$
  - HCF = $2^2 \times 7 = 28$.

Solved Examples

Example 1: Check if 4^n can end with the digit 0 for any natural number n.

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- Solution: For a number to end in 0, it must have 5 as a prime factor. However, $4^n = (2^2)^n = 2^{2n}$. The only prime factor is 2. By the uniqueness of the Fundamental Theorem of Arithmetic, 5 cannot be a factor. Thus, 4^n never ends in 0.

Example 2: Find HCF and LCM of 6 and 20.

- Solution: $6 = 2^1 \times 3^1$; $20 = 2^2 \times 5^1$.
  - HCF(6, 20) = $2^1 = 2$
  - LCM(6, 20) = $2^2 \times 3 \times 5 = 60$

Example 3: Find HCF of 96 and 404, then find their LCM.

- Solution: $96 = 2^5 \times 3$; $404 = 2^2 \times 101$.
  - HCF = $2^2 = 4$.
  - Using formula: LCM $= \frac{96 \times 404}{4} = 9696$.

Example 4: Find HCF and LCM of 6, 72, and 120.

- Solution: $6 = 2 \times$; $72 = 2^3 \times 3^1$; $120 = 2^3 \times 3 \times 5$.
  - HCF = $2 \times 3 = 6$.
  - LCM = $2^3 \times 3^2 \times 5 = 360$.

Example 5: Prove $\sqrt{3}$ is irrational.

- Solution: Assume $\sqrt{3} = a/b$. Squaring gives $3 = a^2/b^2 \Rightarrow a^2 = 3b^2$. So a is divisible by 3. Let $a = 3c$, then $9c^2 = 3b^2 \Rightarrow b^2 = 3c^2$. So b is also divisible by 3. This contradicts that a, b are co-prime.

Example 6: Show $5 - \sqrt{3}$ is irrational.

- Solution: Let $5 - \sqrt{3} = r$ (rational). Then $\sqrt{3} = 5 - r$. Since $5 - r$ is rational but $\sqrt{3}$ is irrational, this is a contradiction. Thus, $5 - \sqrt{3}$ is irrational.

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